Moderne Quantenfeldtheorie und Einführung in das Standardmodell

(exercise sheet 6)

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1. Higgs decay into two photons: $h \to \gamma\gamma$ (3+4+2 points)

One interesting decay channel of the Higgs boson is its decay into two photons. While there is no renormalizable tree-level interaction vertex that couples a Higgs boson to two photons in the SM, the first (lowest-dimensional) non-renormalizable operators (after SSB) are given by

$$\mathcal{L}_{h\gamma\gamma} = c_\gamma \frac{\alpha_e}{6\pi v} h F_{\mu\nu} F^{\mu\nu} + c_{\gamma 5} \frac{\alpha_e}{6\pi v} h F_{\mu\nu} \tilde{F}^{\mu\nu}, \tag{1}$$

where c_γ, $c_{\gamma 5} \in \mathbb{C}$, $\alpha_e \equiv e^2/4\pi$ and $\tilde{F}_{\mu\nu} = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$ (with $\epsilon^{0123} = -1$). The goal of this exercise is to calculate the total unpolarized decay rate for the process $h \to \gamma\gamma$

$$\Gamma(h \to \gamma\gamma) = \frac{1}{2m_h} \int \frac{d^3k_1}{(2\pi)^3\, 2k_1^0} \int \frac{d^3k_2}{(2\pi)^3\, 2k_2^0} (2\pi)^4\, \delta^{(4)}(p - k_1 - k_2) \sum_{\lambda_1=\pm 1} \sum_{\lambda_2=\pm 1} |\mathcal{M}|^2,$$

with respect to the tree-level interactions of the Lagrangian (1). Hereby, (k_1, λ_1) and (k_2, λ_2) are the outgoing momenta and polarizations (only transverse) of the two photons, while p denotes the incoming momentum of the Higgs particle. Note that all external particles are on-shell, i.e. $p^2 = m_h^2$ and $k_1^2 = k_2^2 = 0$.

a) Derive and draw the Feynman rules for both terms in (1), with the convention that all external momenta are flowing into the vertex.

b) Write down the amplitude $\mathcal{M}$ and then calculate the polarization summed and (absolutely) squared amplitude $\sum_{\lambda_1=\pm 1}\sum_{\lambda_1=\pm 1} |\mathcal{M}|^2$. In order to perform the sums, you can use the following completeness relation

$$\sum_{\lambda=\pm 1} \epsilon_\mu(p,\lambda)\, \epsilon_\nu^*(p,\lambda) = -\eta_{\mu\nu} + p_\mu \bar{p}_\nu + p_\nu \bar{p}_\mu\,, \tag{2}$$

where $\bar{p}$ is a null-vector $\bar{p}_\mu \bar{p}^\mu = 0$ and $p_\mu \bar{p}^\mu = -1$. In fact only the first term $-\eta_{\mu\nu}$ can give non-zero contributions since gauge invariance implies that the other two terms vanish when contracted with the corresponding terms in the amplitude. Finally, you should arrive at

$$\sum_{\lambda_1=\pm 1} \sum_{\lambda_2=\pm 1} \mathcal{M}^* \mathcal{M} = 2 \left(\frac{2\alpha_e}{3\pi v}\right)^2 (k_1 \cdot k_2)^2 \left(|c_\gamma|^2 + |c_{\gamma 5}|^2\right). \tag{3}$$

c) The last step is to calculate the decay amplitude, where the Higgs particle is at rest. Performing out all integrations the final result should be given by

$$\Gamma(h \to \gamma\gamma) = \left(\frac{\alpha_e}{4\pi}\right)^2 \frac{m_h^3}{9\pi v^2} \left(|c_\gamma|^2 + |c_{\gamma 5}|^2\right). \tag{4}$$

Please write down how long it took you to solve the exercises.