Moderne Quantenfeldtheorie und Einführung in das Standardmodell

(exercise sheet 5)

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Hand-in on January 7th, 2014

1. Cubic self-interaction $\boldsymbol{WW\gamma}$ (4 points)

Consider the (gauge invariant) kinetic Lagrangian $\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu}$, where $W^a_{\mu\nu}$ denotes the $SU(2)_L$ field strength tensor in the Standard Model. Derive the cubic self-coupling

$$\mathcal{L}_{gauge} \ni \mathcal{L}_{WW\gamma}, \tag{1}$$

expressed in terms of the mass eigenstates $W^\pm_\mu$ for the charged W bosons and A_μ for the photon field. Then determine the Feynman rule for this vertex, with the convention that all momenta are flowing into the vertex, and draw the corresponding diagram.

2. Time-evolution operator (2 points)

Show that the time-ordered exponential (see lecture)

$$U(t,t_0) = T \exp\Big[-i \int_{t_0}^t dt'\, H_I(t') \Big] \tag{2}$$

defined via its Taylor series is indeed a solution of the Schrödinger equation

$$i \frac{\partial}{\partial t} U(t,t_0) = H_I(t)\, U(t,t_0)\ . \tag{3}$$

You may use the following relation ($n \in \mathbb{N}^+$) as given in the lecture

$$\begin{aligned} &\int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \,\dots \int_{t_0}^{t_{n-1}} dt_n\, H_I(t_1)\, H_I(t_2)\, \dots\, H_I(t_n) \\ &\quad = \frac{1}{n!} \int_{t_0}^t dt_1 \int_{t_0}^t dt_2 \,\dots \int_{t_0}^t dt_n\, T\Big\{ H_I(t_1)\, H_I(t_2)\, \dots\, H_I(t_n) \Big\}\ . \end{aligned} \tag{4}$$

3. Discrete Symmetries (3+3+3+1 points)

Consider the action of quantum electrodynamics (QED)

$$S = \int d^4x\, \Big[\bar\psi(x) i D_\mu \gamma^\mu \psi(x) - \frac{1}{4} F_{\mu\nu}(x)\, F^{\mu\nu}(x) \Big], \tag{5}$$

with a Dirac fermion ψ and the $U(1)$ field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. In the following we will check that QED is invariant under the discrete symmetries C and P. Besides the defining relation for the Lorentz algebra you can use that $\gamma^0 \gamma^{\mu\dagger} \gamma^0 = \gamma^\mu$.

a) The parity transformation $x \equiv (t, \vec{x}) \to \tilde{x} \equiv (t, -\vec{x})$ is represented by a unitary operator which transforms a Dirac field operator by

$$P\psi(x)P^\dagger = \eta_P\, \gamma^0\, \psi(\tilde{x}), \qquad\qquad P A^\mu(x) P^\dagger = \mathcal{P}^\mu{}_\nu\, A^\nu(\tilde{x}), \tag{6}$$

where η_P is a complex phase and $(\mathcal{P}^\mu{}_\nu) = \text{diag}(1,-1,-1,-1)$. Show that the QED action is invariant under a parity transformation.

b) Charge conjugation changes particles into antiparticles and is generated by a unitary operator C which transforms the spinor and vector fields according to

$$C\psi(x)C^\dagger = \eta_C\,\mathcal{C}\,\gamma_0^T\psi(x)^*, \qquad\qquad CA^\mu(x)C^\dagger = -A^\mu(x), \tag{7}$$

where η_C is a complex phase and $\mathcal{C}$ is a unitary matrix acting in spinor space fulfilling the relation

$$\mathcal{C}^{-1}\,\gamma_\mu\,\mathcal{C} = -\gamma_\mu^T\ . \tag{8}$$

Show that the QED action is invariant under charge conjugation.

Hint: Treat the fermion spinors as anticommuting fields.

Assuming that the combined transformation CPT is an exact symmetry of quantum electrodynamics, it follows that the QED action is also invariant under time reversal T.

Now we want to switch to the electroweak interactions and consider the neutral and charged currents. Here you may use that γ^0 and γ^2 can be chosen symmetric and that $\gamma_5 = \gamma_5^T = \gamma_5^\dagger$ where $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ as well as $\mathcal{C}\gamma_5 = \gamma_5\,\mathcal{C}$ (e.g., fulfilled in case of Dirac and Weyl representations).

c) Consider the neutral Z-boson current

$$\mathcal{L}_{gauge} \ni \frac{g_2}{\cos\theta_W} Z^\mu \left[\bar{\psi}_L\gamma_\mu T_3\psi_L - \sin^2\theta_W\,\bar{\psi}\gamma_\mu Q\psi\right], \tag{9}$$

and check its transformation behavior under P, C and CP.

Hint: The convention is that $\bar{\psi}_L \equiv (\psi_L)^\dagger\gamma^0 = (P_L\psi)^\dagger\gamma^0$ *with* $P_L = (1-\gamma_5)/2$.

d) Consider now the charged current $W_\mu^\pm$ interaction for three generations

$$\mathcal{L}_{gauge} \ni \frac{g_2}{\sqrt{2}} W_\mu^+\,\bar{u}_L^i V_{\text{CKM}}^{ij}\gamma^\mu d_L^j + \text{h.c.}\,, \tag{10}$$

and perform a CP transformation. What is the condition for V_{CKM} in order to be CP invariant.

Please write down how long it took you to solve the exercises.

Have nice holidays and a good start into the new year !