Impact of Warped Extra Dimensions on the Dipole Coefficients in b to $s\gamma$ Transitions

R. Malm, M. Neubert and C. Schmell

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

1. Introduction

In July 2012 the Higgs boson, the last missing piece of the Standard Model (SM), was discovered at the Large Hadron Collider (LHC) at CERN. Still, the SM faces several theoretical problems it can not explain satisfactorily:

- The Higgs hierarchy problem and the radiative stability
- Hierarchical structure of the Yukawa couplings (flavor puzzle)
- Cosmological constant problem
- Strong CP problem
- ...

A promising possibility to solve the Higgs hierarchy problem and the flavor puzzle is offered by Randall-Sundrum (RS) models [1], in which the SM is embedded in a slice of anti-de Sitter space while the Higgs sector is localized on the "infra-red (IR) brane".

In this work we investigate the flavor-changing neutral current (FCNC) transition $b \to s\gamma$ in the minimal RS model with a brane-localized Higgs sector. The transition is interesting in order to search for new physics since in the SM the dipole Wilson coefficients are Cabibbo-Kobayashi-Maskawa- (CKM) and loop-suppressed.

In order to include the effects of the RS model we implement an effective Lagrangian, in which the heavy Kaluza-Klein (KK) quarks and bosons are integrated out. The most important operators are the electromagnetic dipole operators

$$Q_{7\gamma} = \frac{e\, m_b}{4\pi^2}\, \bar{s}\, \sigma_{\mu\nu} F^{\mu\nu} P_R\, b\,, \qquad \tilde{Q}_{7\gamma} = \frac{e\, m_b}{4\pi^2}\, \bar{s}\, \sigma_{\mu\nu} F^{\mu\nu} P_L\, b\,, \tag{1}$$

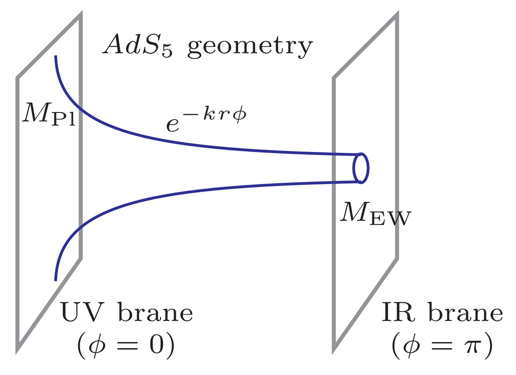
with $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ and the projection operators $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$. Due to operator mixing we also have to consider the chromo-magnetic dipole operators

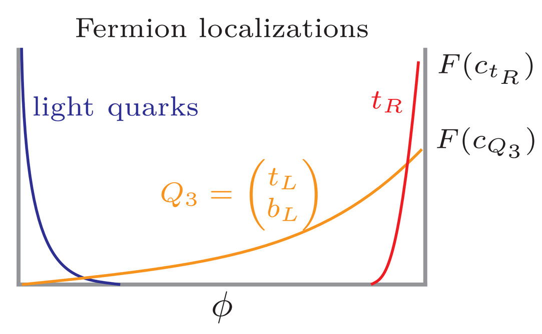
$$Q_{8g} = \frac{g_s m_b}{4\pi^2}\, \bar{s}\, \sigma_{\mu\nu}\, G_a^{\mu\nu} t_a P_R\, b\,, \quad \tilde{Q}_{8g} = \frac{g_s m_b}{4\pi^2}\, \bar{s}\, \sigma_{\mu\nu}\, G_a^{\mu\nu} t_a P_L\, b\,, \tag{2}$$

where t_a are the generators of SU(3)$_c$.

2. Basics of the minimal RS model

- Extra-dimension is a S^1/Z_2 orbifold (coordinate $\phi \in [-\pi, \pi]$)
  - Z_2 parity allows for chiral fermions
- Slice of AdS_5 with metric $ds^2 = e^{-2kr|\phi|}\, \eta_{\mu\nu}\, dx^\mu dx^\nu - r^2 d\phi^2$
  - Curvature and radius are of Planck size $k \sim 1/r \sim M_{\rm Pl}$
  - RS "volume" can be stabilized to $L = kr\pi \approx 33$





- Addressing the Higgs hierarchy problem [1]:

$$\mathcal{L}_{\rm Higgs\ potential} = \sqrt{G}\lambda \left(\mathbf{\Phi}^\dagger \mathbf{\Phi} - v_{5D}^2\right)^2 = \lambda \left(\tilde{\mathbf{\Phi}}^\dagger \tilde{\mathbf{\Phi}} - v_{5D}^2\, e^{-2L}\right)^2$$

  - induced metric at IR brane: $\sqrt{G} = e^{-4L}$
  - Higgs mass $m_h = \sqrt{2\lambda}\, v_{5D}\, e^{-L} \sim M_{\rm Pl} \times e^{-33}$
- Geometrical solution to the flavor puzzle [2–4]:
  - Anarchic 5D Yukawa: $|(\boldsymbol{Y}_q)_{ij}| \sim \mathcal{O}(1)$
  - 5D quark masses: $\mathcal{L}_{\rm mass} = k \sum_i \left(c_{Q_i} \bar{Q}_i Q_i + c_{q_i} \bar{q}_i q_i\right)$
  - Profile overlap with IR brane generates hierarchies, e.g.

$$m_t = \frac{v}{\sqrt{2}} |(Y_u)_{33}||F(c_{Q_3}) F(c_{t_R})|\,,$$

  where $F(c) \approx \sqrt{1+2c}\, e^{-L|1+2c|}$ for $c < -1/2$.
- It is more convenient to use the coordinate $t = \epsilon\, e^{kr|\phi|}$ with $\epsilon = e^{-L}$, such that the UV (IR) brane is localized at $t = \epsilon$ ($t = 1$).

3. Field localizations

Gauge-bosons and fermions

These fields propagate into the extra-dimension and can be decomposed into a zero-mode and a tower of KK modes. For example, the 5D $W^\pm$ vector-boson has the KK representation [5]

$$W_\mu^\pm(x,t) = \frac{1}{\sqrt{r}} \sum_n W_\mu^{\pm(n)}(x)\, \chi_n^W(t)\,, \tag{3}$$

where $W_\mu^{\pm(0)}$ is the 4D $W^\pm$-boson field, $W_\mu^{\pm(n\geq1)}$ are the heavy KK excitations and $\chi_n^W(t)$ are the corresponding wave-functions (profiles) along the extra-dimension. The masses of the lowest KK excitations are set by the KK scale $M_{\rm KK} = k\epsilon \sim$ few TeV.

Higgs field

In this work we focus on the RS model with a brane-localized Higgs field, where the inverse characteristic width of the Higgs field along the extra dimension Δ_h is assumed to be much larger than the inherent ultra-violet (UV) cutoff near the IR brane, i.e. $\Delta_h \gg \Lambda_{\rm TeV} \sim$ several $M_{\rm KK}$.

For consistency reasons we use a regularized δ-function to localize the Higgs field at the IR brane, where we take a square box of width η and height $1/\eta$ such that

$$\delta_h^\eta(t-1) \to \frac{1}{\eta}\, \theta(t-1+\eta)\,, \qquad \text{with} \qquad \eta \ll \frac{v}{M_{\rm KK}}\,. \tag{4}$$

Summary of different localization scenarios for the Higgs boson [6]:

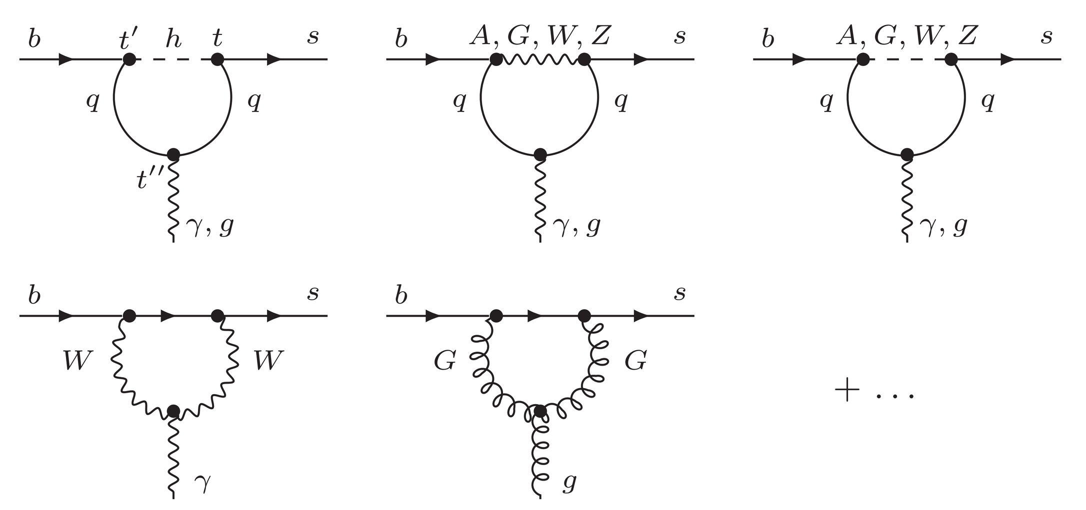
Model	bulk Higgs	narrow bulk-Higgs	brane Higgs
Higgs profile width	$\eta \sim \mathcal{O}(1)$	$\frac{v}{\Lambda_{\rm TeV}} \ll \eta \ll \frac{v}{M_{\rm KK}}$	$\eta \ll \frac{v}{\Lambda_{\rm TeV}}$
Higgs profile	resolved by all modes	resolved by heavy KK modes	not resolved

4. FCNC transition $b \to s\gamma$

Like in the SM, the leading-order contributions to the $b \to s\gamma$ dipole Wilson coefficients in the RS model are loop suppressed. However, corrections to the SM coefficients arise from

- corrections to the SM interaction vertices
- new FCNC couplings of the Higgs and the Z-boson
- and new heavy KK excitations of the bosons and quarks.

Below are shown all relevant Feynman diagrams contributing in a general R_ξ gauge. Dashed lines labeled by A, G, W, Z include the contributions from the scalar component of the 5D gauge bosons and from the corresponding Goldstone scalars (for W, Z) in the Higgs sector.



The Wilson coefficients $C_{7\gamma}$ and $\tilde{C}_{7\gamma}$ are defined via the general parametrization of the transition amplitude

$$\mathcal{A}_{7\gamma} = i\frac{G_F}{\sqrt{2}}\, V_{ts}^* V_{tb} \left[C_{7\gamma}\, \langle s\gamma|Q_{7\gamma}|b\rangle + \tilde{C}_{7\gamma}\, \langle s\gamma|\tilde{Q}_{7\gamma}|b\rangle\right]\,, \tag{5}$$

where G_F is the Fermi constant and V_{ts}, V_{tb} are entries of the CKM matrix. The Wilson coefficients C_{8g} and $\tilde{C}_{8g}$ are defined analogously.

Discussions of the Wilson coefficients for $b \to s\gamma$ in the literature:

- [7–9]: Using naive dimensional arguments (NDA) the authors claim that the dipole Wilson coefficients for a brane Higgs are logarithmically sensitive to the UV cutoff.
- [10]: The authors perform a 5D calculation treating Yukawa interactions as perturbations finding finite dipole Wilson coefficients.
- [11]: The authors perform a 4D calculation, where they focused on the (dominant) diagrams exchanging the first level of KK fermions with intermediate gluons and charged Goldstone bosons.
- [12]: The authors perform a 4D calculation focusing on the diagrams with scalars and KK fermions and claim a non-decoupling effect of the heavy KK Higgses.

5. Calculation in 5D and 4D pictures

In this work we perform a complete calculation of the dipole coefficients including all contributions at one-loop. We derive expressions of the dipole coefficients in the 5D theory using 5D propagators with a full dependence on the Yukawa interactions and show their consistency with the results obtained in the KK decomposed theory.

As an example we discuss the photonic dipole Wilson coefficient for the exchange of the $W^\pm$ vector-boson and two up-type quarks including the zero-modes and their KK excitations

$$\begin{aligned} C_{7\gamma}^W = \frac{2\pi\, Q_u\, \tilde{m}_W^2}{\lambda_t\, \kappa_v} \int_0^\infty dk_E \int_\epsilon^1 dt \int_\epsilon^1 dt'\, B_W(t',t;k_E^2)\, \mathcal{D}_L^{(2)\dagger}(t)\, \boldsymbol{P}_W \\ \Bigg[\left(\frac{11k_E^2}{16}\partial_{k_E} + \frac{5k_E^3}{16}\,\partial_{k_E}^2 + \frac{k_E^4}{48}\,\partial_{k_E}^3\right) \boldsymbol{\Delta}_{LL}^u(t,t';k_E^2)\, \boldsymbol{P}_W\, \mathcal{D}_L^{(3)}(t') \\ + \left(-\frac{3k_E^2}{2}\,\partial_{k_E} - \frac{k_E^3}{2}\,\partial_{k_E}^2\right) \frac{\boldsymbol{\Delta}_{LR}^u(t,t';k_E^2)}{m_b}\, \boldsymbol{P}_W\, \mathcal{D}_R^{(3)}(t')\Bigg]\,, \end{aligned}$$

where $Q_u = 2/3$, $\lambda_t = V_{ts}^* V_{tb}$, $\kappa_v = v/v_{\rm SM}$, $\tilde{m}_W = v g_5/(2\sqrt{2\pi r})$, g_5 is the $SU(2)_L$ 5D gauge-coupling, $\boldsymbol{P}_W = {\rm diag}(1,0)$ and $\mathcal{D}_{L,R}^{(2,3)}(t)$ are the quark profiles of the left and right-chiral strange and bottom quarks.

We can derive closed analytic expressions for the 5D boson propagator function $B_W(t,t';k_E^2)$ and the 5D quark propagator functions $\boldsymbol{\Delta}_{LL}^u(t,t';k_E^2)$ and $\boldsymbol{\Delta}_{LR}^u(t,t';k_E^2)$ in the broken electroweak phase, i.e. with non-trivial boundary conditions at the IR brane.

As a cross-check we have also performed a calculation in the KK decomposed theory, in which the propagators take the form

$$\begin{aligned} B_W(t,t';k_E^2) &= \sum_{n=0}^\infty \frac{\chi_n^W(t)\, \chi_n^W(t')}{m_{W_n}^2 + k_E^2 - i0}\,, \\ \boldsymbol{\Delta}_{LL}^u(t,t';k_E^2) &= \sum_{n=0}^\infty \frac{\mathcal{U}_L^{(n)}(t)\, \mathcal{U}_L^{(n)\dagger}(t')}{-k_E^2 - m_{u_n}^2 + i0}\,, \end{aligned} \tag{6}$$

where $\mathcal{U}_L^{(n)}(t)$ are the up-type left-chiral quark profiles. In the KK decomposed theory $C_{7\gamma}^W$ is given by

$$C_{7\gamma}^W = \frac{-Q_u}{\lambda_t \kappa_v^2} \sum_{m,n} \frac{\tilde{m}_W^2}{m_{W_m}^2} \left[\frac{m_{u_n}}{m_b} I_1(x_m^n) V_{2mn}^{W^-} \tilde{V}_{nm3}^{W^+} + I_2(x_m^n) V_{2mn}^{W^-} V_{nm3}^{W^+}\right],$$

where $I_{1,2}(x_m^n)$ are loop functions with $x_m^n = m_{u_n}^2/m_{W_m}^2$, and $V_{nmk}^{W^\pm}$, $\tilde{V}_{nmk}^{W^\pm}$ are overlap integrals.

4. Analysis of the Wilson coefficients

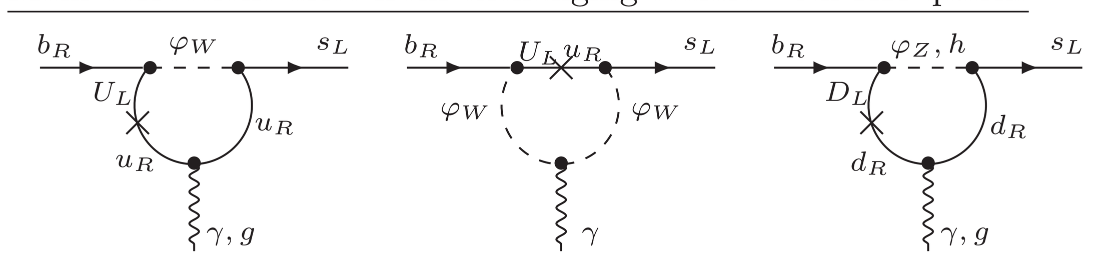
Finiteness of the integrals

We derive the ultra-violet behavior of the 5D propagator functions, e.g. the W-boson propagator behaves like

$$B_W(t,t';k_E^2) \sim \frac{\sqrt{tt'}}{k_E} \exp\left(-\frac{k_E}{M_{\rm KK}}\,|t-t'|\right)\,, \tag{7}$$

for large Euclidean momenta $k_E/M_{\rm KK} \gg 1/t, 1/t'$. Integrating (7) along t, t' shows that the propagator falls off with k_E^{-2}. This property can be used to show the finiteness of the Wilson coefficients, where we note that scalar diagrams with brane-localized vertices have to be analyzed using the regularized δ-function (4).

Main corrections arise from exchanging scalars with KK quarks



Goldstone bosons are denoted by φ_W, φ_Z and the Higgs boson by h. Internal solid lines u_R denote the exchange of singlet up-type KK quarks, while U_L, D_L imply the exchange of $SU(2)_L$ doublet KK quarks. To leading order in $v^2/M_{\rm KK}^2$ we obtain

$$\begin{aligned} C_{7\gamma}^{W,{\rm KK}} &\approx -\frac{(Q_u+1)}{8\lambda_t}\Bigg[\frac{v}{\sqrt{2}m_b}\frac{v^2}{M_{\rm KK}^2}\mathcal{D}_L^{(2)\dagger}(1^-)\boldsymbol{P}_{12}\boldsymbol{Y}_u\boldsymbol{Y}_u^\dagger\boldsymbol{Y}_d\mathcal{D}_R^{(3)}(1^-) \\ &\qquad - \frac{2\tilde{m}_W^2}{m_W^2}\frac{m_t}{m_b}\, V_{203}^{W^-}\, \tilde{V}_{303}^{W^+}\Bigg]\,, \\ C_{7\gamma}^{h+Z,{\rm KK}} &\approx c \times \frac{Q_d}{2\lambda_t}\frac{v}{\sqrt{2}m_b}\frac{v^2}{M_{\rm KK}^2}\mathcal{D}_L^{(2)\dagger}(1^-)\boldsymbol{P}_{12}\boldsymbol{Y}_d\boldsymbol{Y}_d^\dagger\boldsymbol{Y}_d\mathcal{D}_R^{(3)}(1^-)\,, \end{aligned}$$

where we combined the Higgs and the Z-Goldstone boson contributions with $c = 0$ in the brane-localized Higgs scenario and $c = -1/12$ in the narrow bulk-Higgs scenario.

RG running to the meson scale $\mu_b \sim m_b$

We distinguish corrections from the exchange of zero modes $C_i^{\rm RS,0}(\mu_W)$ at the electroweak scale $\mu_W \sim m_W$ and of virtual KK particles $C_i^{\rm RS,KK}(\mu_{\rm KK})$ at the KK scale $\mu_{\rm KK} \sim M_{\rm KK}$ such that

$$\begin{aligned} C_{7\gamma}^{\rm RS}(\mu_b) = 0.51\, C_{7\gamma}^{\rm RS,KK}(\mu_{\rm KK}) + 0.12\, C_{8g}^{\rm RS,KK}(\mu_{\rm KK}) + 0.63\, C_{7\gamma}^{\rm RS,0}(\mu_W) \\ + 0.10\, C_{8g}^{\rm RS,0}(\mu_W) - 0.19\, C_2^{\rm RS,0}(\mu_W)\,. \end{aligned}$$

We also include corrections to the charged current-current operator $Q_2 = (\bar{s}_j \gamma_\mu P_L c_j)(\bar{c}_i \gamma^\mu P_L b_i)$.

7. Summary

- We calculated the radiative Wilson coefficients $C_{7\gamma,8g}$ and $\tilde{C}_{7\gamma,8g}$ in the context of the minimal RS model with a Higgs sector localized on the IR brane using the 5D approach, where the coefficients are expressed in terms of integrals of 5D propagators. Since we kept the full dependence on the Yukawa matrices, the integral expressions are formally valid to all orders in $v^2/M_{\rm KK}^2$.
- In addition we related our results to the expressions obtained in the Kaluza-Klein decomposed theory and showed the consistency in both pictures analytically and numerically, which presents a non-trivial cross-check.
- We demonstrated the finiteness of the dipole Wilson coefficients.
- The dominant KK corrections in RS are given by scalar penguin diagrams with a chirality flip on the internal KK quark lines. The exchange of KK gluons and photons turns out to be subleading, due to an approximate flavor alignment.
- The dipole Wilson coefficients for the scalar penguin diagrams are "model-dependent". For a brane-localized Higgs sector the Higgs and the Z-boson contributions cancel to good approximation.
- Corrections to $C_{7\gamma}(\mu_b)$ and $C_{8g}(\mu_b)$ with respect to the SM lie in the few % region (for anarchiy Yukawa matrices $(\boldsymbol{Y}_q)_{ij} \leq 3$). Corrections to the chirality flipped Wilson coefficients are larger since they are not suppressed by m_s/m_b as it is the case in the SM.

References

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[2] Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000) [hep-ph/9912408].

[3] T. Gherghetta and A. Pomarol, Nucl. Phys. B **586**, 141 (2000) [hep-ph/0003129].

[4] S. J. Huber and Q. Shafi, Phys. Lett. B **498**, 256 (2001) [hep-ph/0010195].

[5] S. Casagrande, F. Goertz, U. Haisch, M. Neubert and T. Pfoh, JHEP **0810**, 094 (2008) [arXiv:0807.4937 [hep-ph]].

[6] R. Malm, M. Neubert, K. Novotny and C. Schmell, arXiv:1303.5702 [hep-ph].

[7] K. Agashe, G. Perez and A. Soni, Phys. Rev. Lett. **93**, 201804 (2004) [hep-ph/0406101].

[8] K. Agashe, G. Perez and A. Soni, Phys. Rev. D **71**, 016002 (2005) [hep-ph/0408134].

[9] K. Agashe, A. E. Blechman and F. Petriello, Phys. Rev. D **74**, 053011 (2006) [hep-ph/0606021].

[10] M. Blanke, B. Shakya, P. Tanedo and Y. Tsai, JHEP **1208**, 038 (2012) [arXiv:1203.6650 [hep-ph]].

[11] P. Biancofiore, P. Colangelo and F. De Fazio, Phys. Rev. D **89**, no. 9, 095018 (2014) [arXiv:1403.2944 [hep-ph]].

[12] K. Agashe, A. Azatov, Y. Cui, L. Randall and M. Son, arXiv:1412.6468 [hep-ph].