Solution to the RS Flavor Problem

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1. The Randall-Sundrum Model

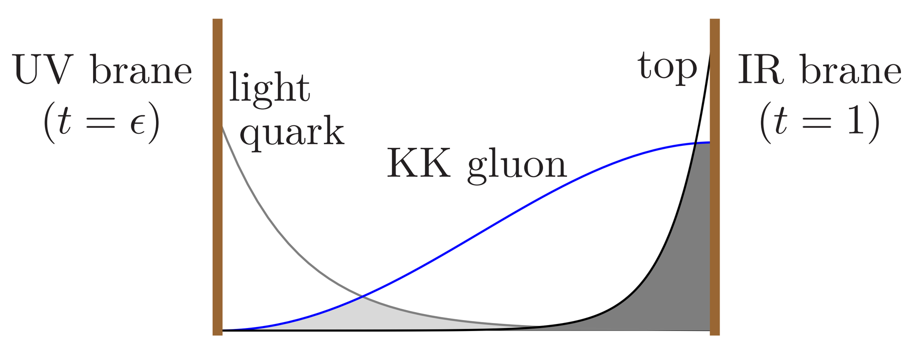
Proposed in 1999 by Lisa Randall and Raman Sundrum [1], the Randall-Sundrum (RS) Model is a compelling candidate for an extension of the Standard Model (SM). Assuming in addition to four-dimensional space-time one warped and compact extra dimension, parametrized by the coordinate $t \in [\epsilon, 1]$ with $\epsilon \equiv M_{\rm EW}/M_{\rm Pl}$, it can explain the Higgs hierarchy problem as well as the flavor puzzle in terms of a geometrical mechanism. Given the metric of the complete space,

$$ds^2 = \frac{\epsilon^2}{t^2}\Big(\eta_{\mu\nu}dx^\mu dx^\nu - \frac{1}{M_{\rm KK}^2}dt^2\Big),$$

the warp factor ϵ/t rescales the units of length and mass in such a way that the fundamental Planck scale $M_{\rm Pl}$ is warped down from 10^{19} GeV to the TeV scale as t varies between ϵ (UV brane) and 1 (IR brane). As a consequence, the Higgs mass is stabilized around the electroweak scale as long as the Higgs sector is localized near the IR brane. All other particles of the SM are extended to five-dimensional fields, that are allowed to propagate in the extra-dimension and which can be decomposed into a sum of 4D modes times profile functions that only depend on the coordinate t. For instance, the 5D gluon field can be expressed by

$$G_\mu(x,t) \propto \sum_{n=0}^{\infty} G_\mu^{(n)}(x)\,\chi_n^G(t),$$

where $\chi_n^G(t)$ is the profile function of the n-th gluon mode. While the SM gluon is represented by $G_\mu^{(0)}$ and corresponds to a flat profile, higher massive modes have non-flat profiles located towards the IR brane and are referred to as Kaluza-Klein (KK) excitations.



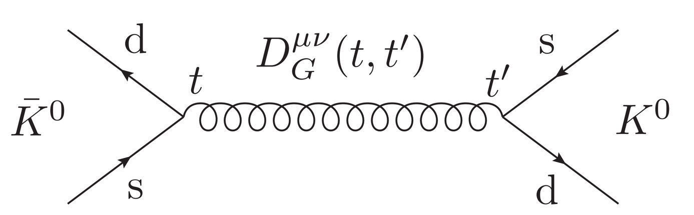
Their couplings to SM quarks are governed by the profile overlap which results in a suppression of tree-level flavor-changing neutral current (FCNC) processes, due to the different profile localizations. This so-called RS GIM mechanism has proven to be very effective in suppressing FCNC transitions for a lot of observables, but there is one single exception:

$$\epsilon_K^{\rm RS} \propto {\rm Im}\langle K^0|\mathcal{H}_{\rm eff}^{\rm RS}|\bar K^0\rangle,$$

which measures indirect CP violation in K^0-$\bar K^0$ mixing. In fact, the comparison with the experimental value $\epsilon_K^{\rm exp}$ pushes the intrinsic scale $M_{\rm KK}$ to large values in the 6-7 TeV range, reintroducing a hierarchy between $M_{\rm KK}$ and the electroweak scale and thus spoiling the benefit of the RS Model.

2. Observable ϵ_K in the RS Model

While the theoretical prediction for ϵ_K in the SM is loop-suppressed (box diagrams), the RS Model already allows for flavor-changing diagrams at tree level (s- and t-channel) by exchanging massive gluon, photon, Z modes or the Higgs [2],



but where the main contribution is given by the gluon excitations. Summing up the 4D propagators of all infinitely many gluon modes corresponds to the calculation of the 5D gluon propagator (in the limit of vanishing momenta $p \to 0$)

$$D_G^{\mu\nu}(t,t') = \frac{\eta^{\mu\nu}L}{4\pi r M_{\rm KK}^2}\left[t_<^2 + \frac{1}{2L^2} - \frac{t^2}{L}\left(\frac{1}{2} - \ln t\right) - \frac{t'^2}{L}\left(\frac{1}{2} - \ln t'\right)\right]$$

where only the $t_<^2 \equiv \min[t^2, t'^2]$ term in the bracket contributes to the $\Delta S = 2$ process. Choosing the following basis of four-quark operators

$$Q_1 = (\bar d_L\gamma^\mu s_L)(\bar d_L\gamma_\mu s_L), \qquad Q_4 = (\bar d_R s_L)(\bar d_R s_L),$$
$$\tilde Q_1 = (\bar d_R\gamma^\mu s_R)(\bar d_R\gamma_\mu s_R), \qquad Q_5 = (\bar d_R^\alpha s_L^\beta)(\bar d_R^\beta s_L^\alpha),$$

one obtains contributions for the Wilson coefficients of equal chirality $C_1^{\rm RS}$ and $\tilde C_1^{\rm RS}$ as well as for the mixed chirality coefficients $C_4^{\rm RS}$ and $C_5^{\rm RS}$. While all these coefficients are of the same order $\sim 1/M_{\rm KK}^2$, the renormalization group running from the high KK scale down to the hadronic scale and the multiplication with the operator matrix elements introduce a relative weighting (roughly)

$$\langle K^0|\mathcal{H}_{\rm eff}^{\rm RS}|\bar K^0\rangle \propto C_1^{\rm RS} + \tilde C_1^{\rm RS} + 137\left(C_4^{\rm RS} + \frac{1}{N_c}C_5^{\rm RS}\right),$$

such that the main contributions arise from $C_4^{\rm RS}$ and $C_5^{\rm RS}$.

3. Idea: Extension of the Strong Gauge Group

In order to minimize the contributions to the mixed chirality coefficients $C_4^{\rm RS}$ and $C_5^{\rm RS}$, one can replace the color gauge group by [3]

$$G_c \equiv SU(3)_D \times SU(3)_S,$$

where $SU(2)_L$ doublet fields Q transform as triplets under $SU(3)_D$ and as singlets under $SU(3)_S$. Reversely, the $SU(2)_L$ singlet fields q^c transform as triplets under $SU(3)_S$ but are invariant under $SU(3)_D$. With respect to the gauge fields $G_D^{a,\mu}$ and $G_S^{a,\mu}$ of the enlarged group G_c, the coupling to quarks is then given by

$$\mathcal{L}_{\rm int} \ni g_D\,\bar Q\,G_D^{a,\mu}\gamma_\mu T^a Q + g_S\,\bar q^c\,G_S^{a,\mu}\gamma_\mu T^a q^c\,.$$

In the mass eigenbasis, one recovers the gluon $G^\mu = G_D^\mu\cos\vartheta + G_S^\mu\sin\vartheta$ with $\tan\vartheta = g_D/g_S$ and the orthogonal combination $\mathcal{A}^\mu = -G_D^\mu\sin\vartheta + G_S^\mu\cos\vartheta$, which is referred to as the pseudo-axial gluon. The interaction Lagrangian then becomes

$$\mathcal{L}_{\rm int} \ni g_s\left(\bar Q\,G_\mu^a\gamma^\mu T^a Q + \bar q^c\,G_\mu^a\gamma^\mu T^a q^c\right) + g_s\left(-\tan\vartheta\,\bar Q\,\mathcal{A}_\mu^a\gamma^\mu T^a Q + \cot\vartheta\,\bar q^c\,\mathcal{A}_\mu^a\gamma^\mu T^a q^c\right), \quad (1)$$

where $g_s = g_D\cos\theta = g_S\sin\theta$ is the strong coupling, so that in case of the mixing angle $\vartheta = 45°$ the field $\mathcal{A}_\mu$ couples axially to quarks.

Given that in the 4D effective theory the quark doublets will be decomposed into left-handed and the singlets into right-handed quarks, the contributions of the gluon and of the pseudo-axial gluon KK modes to the relevant mixed-chirality four quark operators have opposite signs (independent of the mixing angle ϑ), thus leading to a cancellation at leading order.

4. Pseudo-Axial Gluon: Boundary Conditions & 5D Propagator

The properties of the pseudo-axial gluon are determined by its boundary conditions (BCs) at the UV and IR brane

$$\partial_t\chi_n^{\mathcal{A}}(t)\big|_{t=\epsilon^+} = b_\epsilon\,\chi_n^{\mathcal{A}}(\epsilon^+), \qquad \partial_t\chi_n^{\mathcal{A}}(t)\big|_{t=1^-} = -b_1\,\chi_n^{\mathcal{A}}(1^-),$$

with a priori general parameters b_ϵ and b_1. The Neumann-Neumann (NN) case with $b_\epsilon = b_1 = 0$ would imply a massless zero-mode and is consequently discarded. The breaking of G_c to its diagonal subgroup enforces mixed (M), $b_1 \neq 0$, boundary conditions at the IR brane, while experimental constraints also lead to $b_\epsilon \gtrsim \epsilon$. Since each BC is generated by spontaneous symmetry breaking (SSB) of a physical Higgs sector at the UV and IR brane, one expects

$$b_\epsilon \sim \frac{v_{\rm UV}^2}{M_{\rm KK}^2}, \qquad b_1 \sim \frac{v_{\rm IR}^2}{M_{\rm KK}^2},$$

with natural vacuum expectation values (VEVs) $v_{\rm UV} \sim M_{\rm Pl}$ and $v_{\rm IR} \sim v = 246\,$GeV. By means of all constraints, one can express the 5D propagator of the pseudo-axial gluon (in the limit $p \to 0$) as

$$D_{\mathcal{A}}^{\mu\nu}(t,t') = \frac{\eta^{\mu\nu}L}{4\pi r M_{\rm KK}^2}\left[t_<^2 - \frac{b_1}{2+b_1}t^2t'^2\right], \quad (2)$$

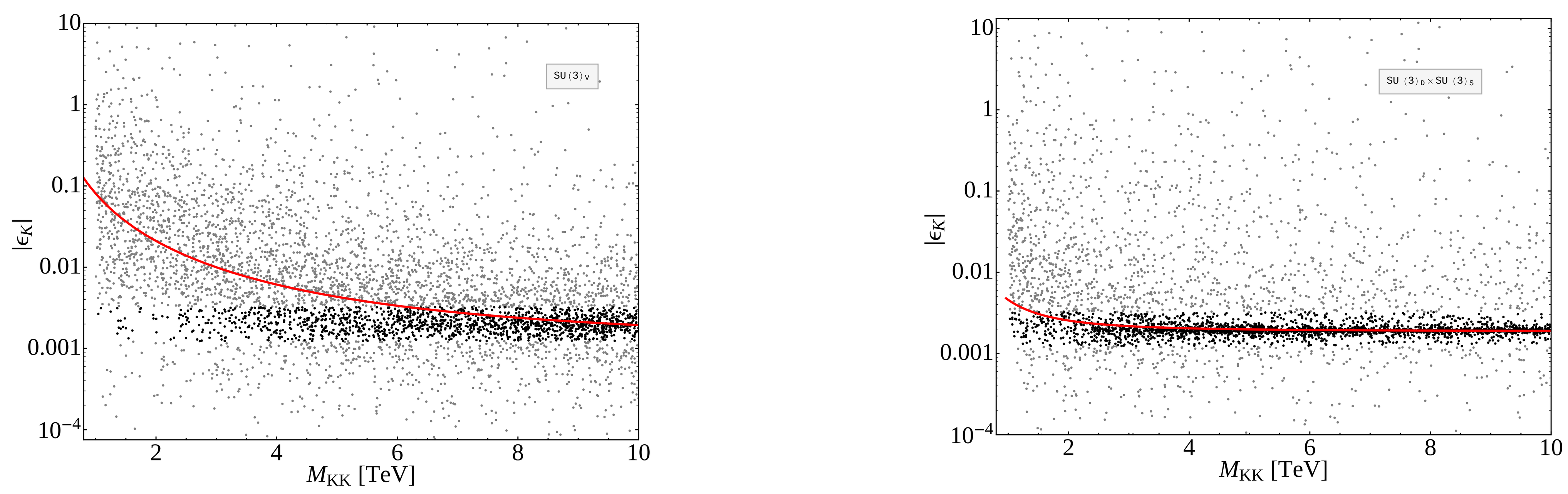
whose first term $\propto t_<^2$ in the bracket does also appear in the gluon propagator. Due to the relative sign in (1), this exactly cancels the mixed chirality contributions from the gluon. This leaves only the term $\propto t^2t'^2$ whose size is determined by the suppressed coefficient b_1. Therefore the combined Wilson coefficients are of order

$$C_1^{\rm RS},\,\tilde C_1^{\rm RS} \sim \frac{1}{M_{\rm KK}^2}, \qquad C_4^{\rm RS},\,C_5^{\rm RS} \sim \frac{v^2}{M_{\rm KK}^4}, \quad (3)$$

showing that $C_4^{\rm RS}$ and $C_5^{\rm RS}$ are smaller by a factor of $\sim 10^{-2}$, compared to the RS Model without the pseudo-axial gluon.

5. Results: ϵ_K in the RS Model and in the Extended Version

The two scatter plots below show the distribution of 5000 physical parameter points in the RS Model (left panel) and in the extended version of it (right panel). Each of these points represents a physical setup that reproduces the correct quark masses and CKM matrix entries. Furthermore, points are shaded black if they lead to ϵ_K values compatible with experiment. The red line represents the median curve of all points, which crosses the black "band" roughly at the $M_{\rm KK}$ value that is preferred without fine-tuning. It becomes evident from the plots, that the extended model allows for a $M_{\rm KK}$ value in the range of 1-2 TeV compared to 6-7 TeV in the original model.



6. Extension of the Higgs Sector in the RS Model

The solution to the RS flavor problem relies on the (MM) boundary conditions for the pseudo-axial gluon. To generate such BCs, one needs additional scalar fields localized at the UV and IR brane, that are also charged under the symmetry group G_c. The aim is then to find a realistic and minimal extension of the Higgs sector in the RS Model, that spontaneously breaks the complete gauge group to the unbroken color $SU(3)_c$ and electromagnetic $U(1)_{Q_e}$,

$$G_c \times SU(2)_L \times U(1)_Y \xrightarrow{\rm SSB} SU(3)_c \times U(1)_{Q_e}\,.$$

The minimal Higgs sector contains one scalar field S at the UV brane and three scalar fields at the IR brane, where H_u and H_d allow for Yukawa terms for the up- and down-type quarks while H_l is needed to prevent physical (massless) Nambu-Goldstone bosons. Their transformation behavior under the complete gauge group as well as their localization and assumed VEVs are listed in the following table. After spontaneous symmetry-breakdown, one obtains new neutral and charged $SU(3)_c$ singlet as well as octet fields and one can determine the BCs of the pseudo-axial gluon from a matching calculation. They are given by

scalars	$SU(3)_D$	$SU(3)_S$	$SU(2)_L$	$U(1)_Y$	Brane	VEV
S	**3**	**3***	**1**	0	UV	v_S
H_l	**1**	**1**	**2**	1/2	IR	v_l
H_u	**3**	**3***	**2**	$-1/2$	IR	v_u
H_d	**3**	**3***	**2**	1/2	IR	v_d

$$v_{\rm UV}^2 \sim v_S^2, \qquad v_{\rm IR}^2 \sim v_u^2 + v_d^2, \quad (4)$$

where $v_S \sim M_{\rm Pl}$ is the natural VEV at the UV brane. In fact, the VEVs at the IR brane are constrained by reproducing the correct $W^\pm$ and Z boson masses, which lead to the relation

$$v_u^2 + v_d^2 + v_l^2 = (246\,{\rm GeV})^2, \quad (5)$$

and confirms the assumed small value for $b_1 \ll 1$. This constraint does also affect the localization of the quark profiles by shifting them towards the IR brane.

Prospective Work

- Constructing a physical implementation of the scalar sector at the UV and IR brane.
- Calculation of the effects on ϵ_K, due to the new quark localizations and the scalars in the extended model.

References

[1] L. Randall and R. Sundrum. Phys.Rev.Lett. **83** (1999) 3370–3373.

[2] M. Bauer, S. Casagrande, U. Haisch, and M. Neubert. JHEP **1009** (2010) 017.

[3] M. Bauer, R. Malm, and M. Neubert. Phys.Rev.Lett. **108** (2012) 081603.