Loop Processes and Higgs Phenomenology in a Warped Extra Dimension

based on [RM,Neubert,Novotny,Schmell:hep-ph/1303.5702]
[Hahn,Hörner,RM,Neubert,Novotny,Schmell:hep-ph/1312.5731]
[RM,Neubert,Schmell:hep-ph/1408.4456]

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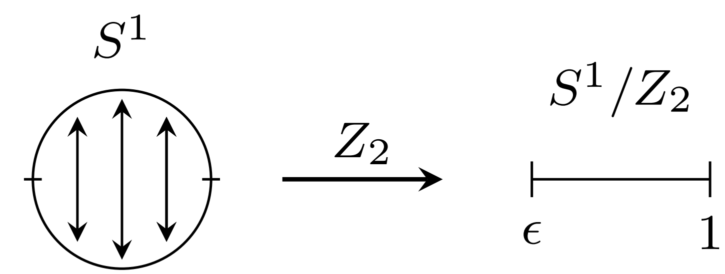
Outline

- Introduction to the Randall-Sundrum model
- Loops in a warped extra dimension with a focus on Higgs physics
  - $gg \to h$
  - $h \to \gamma\gamma$
- Implications for the RS parameter space

One warped extra dimension - Basics

Setup

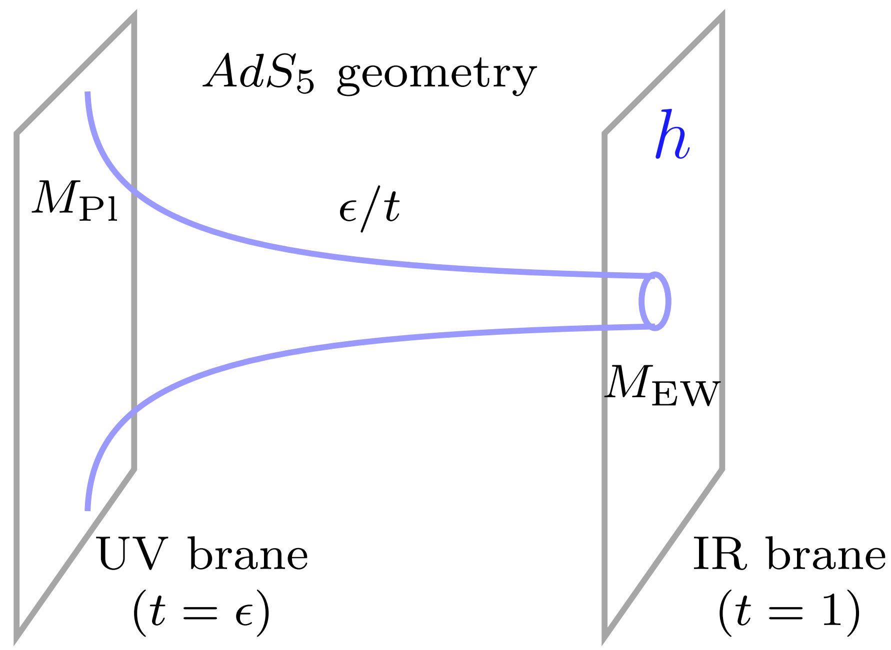
[Randall,Sundrum:hep-ph/9905221]



- extra dimension: S^1/Z_2 orbifold $(t \in [\epsilon, 1])$
  - Z_2 parity: allows for chiral fermions in the spectrum via BCs

$$\partial_t \, \Psi(x,t)|_{t=\epsilon,1} = b_t \, \Psi(x,t)|_{t=\epsilon,1}$$

- 5D space-time = slice of AdS_5 bounded by two 3-branes
  - non-factorizable metric: $ds^2 = \frac{\epsilon^2}{t^2}\left(dx^\mu dx_\mu - \frac{1}{k^2\epsilon^2}\, dt^2\right)$
  - Ricci-scalar (negative scalar curvature): $R = -20k^2 \quad \text{with} \quad k \sim M_{\rm Pl}$
  - radius of S^1 : $r \sim M_{\rm Pl}^{-1}$
  - warp factor rescaling energy/length: $\epsilon/t \quad \text{with} \quad \epsilon = e^{-kr\pi}$
  - electroweak hierarchy: $kr \approx 11 \;\rightarrow\; \epsilon \approx \frac{M_{\rm EW}}{M_{\rm Pl}} \approx 10^{-16}$



- particle content
  - Higgs field is restricted to be on or near the IR brane
  - Kaluza-Klein (KK) decomposition of 5D bulk fields: $\Phi(x,t) \sim \phi_0(x)\,\chi_0(t) + \sum_{n=1}^{\infty} \phi_n(x)\,\chi_n(t)$
  - masses of the vector-like KK excitations: $m_n \sim n\,\pi\, M_{\rm KK} \quad \text{with} \quad \boxed{M_{\rm KK} = k\epsilon \sim \text{few TeV}}$
- expansion parameter for new physics effects: $\frac{v}{M_{\rm KK}}$

Why is this model interesting ?

Higgs hierarchy puzzle — Why is $m_h^2 \sim 10^{-32}\, M_{\rm Pl}^2$? Why is the Higgs mass radiatively stable ?

$$S_{\rm IR} \ni \int d^4x \int_\epsilon^1 \frac{dt}{t}\, \sqrt{G}\, \delta(t-1)\, \lambda \left(|\Phi|^2 - v_5^2\right)^2 = \int d^4x\, \epsilon^4 \lambda(|\Phi|^2 - v_5^2)^2 = \int d^4 x\, \lambda \left(|\tilde{\Phi}|^2 - (\epsilon v_5)^2\right)^2$$

- induced metric at IR brane: $\sqrt{G} = \epsilon^4$
- 4D vev: $v = \epsilon v_5 \approx 246\ \text{GeV}$ (little hierarchy remains)

Fermion hierarchy puzzle — What explains the hierarchy in the Yukawa matrices / fermion masses ?

- fermion 5D bulk-mass terms allowed (gauge invariant)

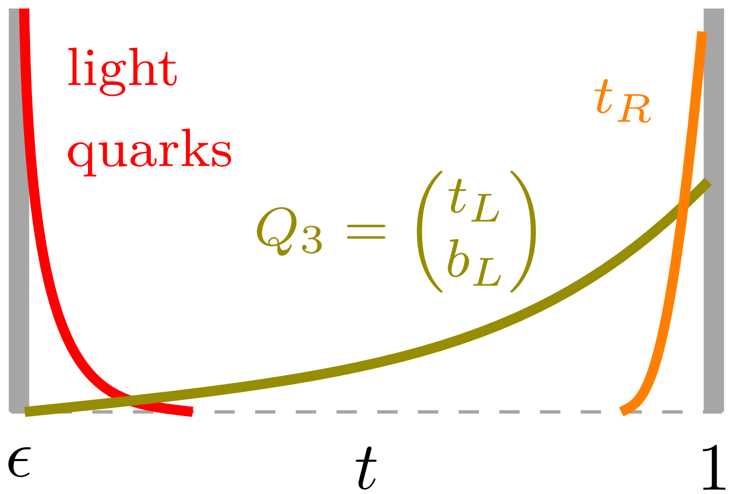
$$\mathcal{L}_{\rm mass} \ni k\,(\bar{Q}\boldsymbol{c}_Q Q + \bar{q}\boldsymbol{c}_q q) \quad \text{with} \quad c_{Q_i}, c_{q_i} \sim \mathcal{O}(1)$$

- fermion masses given by, e.g.

$$m_t = \frac{v}{\sqrt{2}} |(\boldsymbol{Y}_u)_{33}||F(c_{Q_3})F(c_{q_3})| \quad \text{with} \quad (\boldsymbol{Y}_q)_{ij} \sim \mathcal{O}(1)$$

  - profile overlap with IR brane generates hierarchies

$$F(c) \approx \sqrt{|1+2c|} \times \begin{cases} 1 & , c > -1/2 \\ \epsilon^{|1+2c|} & , c < -1/2 \end{cases}$$



Constraints on the model

- direct detection of KK gluon resonances: $M_{g^{(1)}} > 2\,\text{TeV}$ $M_{g^{(1)}} = 2.45\,M_{\text{KK}}$

Minimal RS model

- based on SM gauge group
- tension with $Zb_L\bar{b}_L$ vertex and electroweak S,T parameters: $M_{g^{(1)}} > 12.3\,\text{TeV at } 95\%\ \text{CL}$

Custodial RS model

- implement enlarged bulk gauge group: $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times P_{LR}$
- protect T parameter by a remaining custodial symmetry on the IR brane $SU(2)_L \times SU(2)_R \to SU(2)_V$
- extended quark sector (Z_2 even fields) $Q_L \sim (\mathbf{2},\mathbf{2})_{\frac{2}{3}}\,,\ u_R^c \sim (\mathbf{1},\mathbf{1})_{\frac{2}{3}}\,,\ \mathcal{T}_R \sim (\mathbf{3},\mathbf{1})_{\frac{2}{3}} \oplus (\mathbf{1},\mathbf{3})_{\frac{2}{3}}$
    - 15 quark excitations in up-type sector (per KK level)
    - 9 quark excitations in down-type sector (per KK level)
    - 9 exotic fermion excitations with electric charge 5/3 (per KK level)

- tension with electroweak S,T parameter: $M_{g^{(1)}} > 4.8\,\text{TeV at } 95\%\ \text{CL}$

Loops in a warped extra dimension

- Randall-Sundrum models are effective field theories with an unknown UV completion
  - e.g. gauge coupling of QED in 5D: $\mathcal{L}_{5D} \ni e_5 \, \bar{\Psi} \not{A} \Psi \quad \text{with} \quad [\Psi] = E^2, \, [A_\mu] = E^{\frac{3}{2}} \quad \rightarrow \quad [e_5] = E^{-\frac{1}{2}}$
  - position dependent ultra-violet 4D cutoff: $\Lambda_{\mathrm{UV}}(t) = \frac{\epsilon}{t} \, M_{\mathrm{Pl}}$ $\qquad$ IR brane: $\Lambda_{\mathrm{UV}}(1) \sim 10 \, M_{\mathrm{KK}}$

- Calculation of ultra-violet finite one-loop processes in RS models with Higgs sector on IR brane
  - lepton flavour violation:

    $\mu \to e\gamma$ [Agashe,Blechmann,Petriello:hep-ph/0606021] [Csaki,Grossman, Tanedo,Tsai:hep-ph/1004.2037]

    $(g-2)_\mu$ [Davoudiasl,Hewett,Rizzo:hep-ph/0006097] [Beneke,Dey,Rohrwild:hep-ph/1209.5897] [Beneke,Moch,Rohrwild:hep-ph/1404.7157]

  - quark flavour violation:

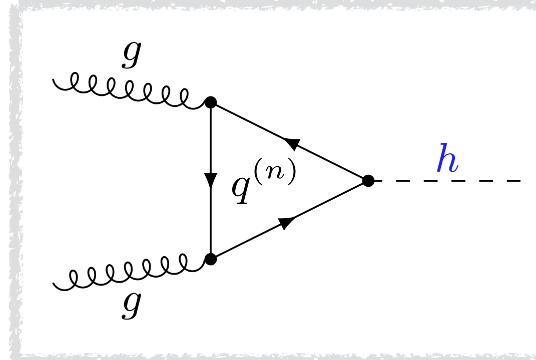
    $b \to s\gamma$ [Gedalia,Isidori,Perez:hep-ph/0905.3261] [Blanke,Shakya,Tanedo,Tsai:hep-ph/1203.6650]
    [Delaunay,Kamenik,Perez,Randall:hep-ph/1207.0474]

  - loop-induced Higgs couplings:

    $gg \to h$ [Casagrande,Goertz,Haisch,Neubert,Pfoh:hep-ph/1005.4315] [Azatov,Toharia,Zhu:hep-th/1006.5939]
    [Carena,Casagrande,Goertz,Haisch,Neubert:hep-th/1204.0008] [RM,Neubert,Novotny,Schmell:hep-ph/1303.5702]
    [Frank,Pourtolami,Toharia:hep-ph/1311.1824]

    $h \to \gamma\gamma$ [Casagrande,Goertz,Haisch,Neubert,Pfoh:hep-ph/1005.4315] [Azatov,Toharia,Zhu:hep-th/1006.5939]
    [Hahn,Hörner,RM,Neubert,Novotny,Schmell:hep-ph/1312.5731]

Loops in Higgs physics



General procedure to calculate $gg \to h,\ h \to \gamma\gamma$

- use 5D propagators for internal lines in mixed position-momentum space e.g. 5D W-boson vector propagator (Feynman 't Hooft gauge)

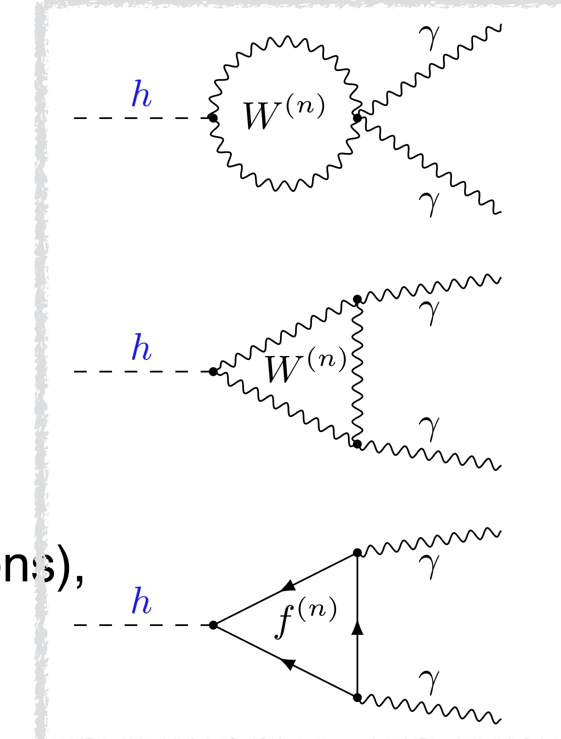
$$ig^{\mu\nu} B_W(t,t';-p^2) \equiv ig^{\mu\nu} \sum_{n=0}^{\infty} \frac{\chi_n^W(t)\,\chi_n^W(t')}{m_{W_n}^2 - p^2}$$

  - advantage: compact analytic expressions can be derived

$$B_W(t,t';-p^2) = \frac{Ltt'}{4M_{\rm KK}^2} \frac{\left[\hat{p}D_{10}(t_>,1) - b_1 D_{11}(t_>,1)\right] D_{10}(t_<,\epsilon)}{\hat{p}D_{00}(1,\epsilon) - b_1 D_{10}(1,\epsilon)}$$

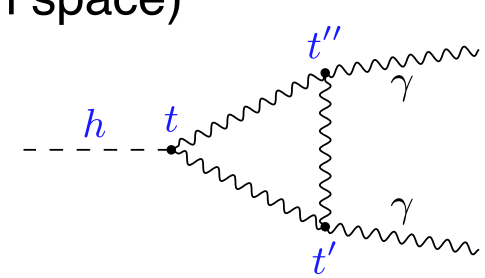
$$\text{with} \quad D_{ij}(t,t') = J_i(\hat{p}t)\,Y_j(\hat{p}t') - Y_i(\hat{p}t)\,J_j(\hat{p}t'),\ \hat{p} = \frac{p}{M_{\rm KK}},\ t_> = \mathrm{Max}(t,t')$$

  - we are working in the broken electroweak phase (non-trivial IR boundary conditions), which allows for our results to be valid to all all orders in $v^2/M_{\rm KK}^2$



- vertices with SM gluons/photons are KK-number diagonal, i.e. (in euclidean momentum space)

$$\int dt''\, B_W(t,t'';p_E^2)\, B_W(t'',t';p_E^2) = -\frac{\partial}{\partial p_E^2} B_W(t,t',p_E^2)\,,$$



  - loop integrands involve only one 5D propagator
  - remaining coordinate is evaluated at the IR brane t = 1 (where the Higgs is localised)

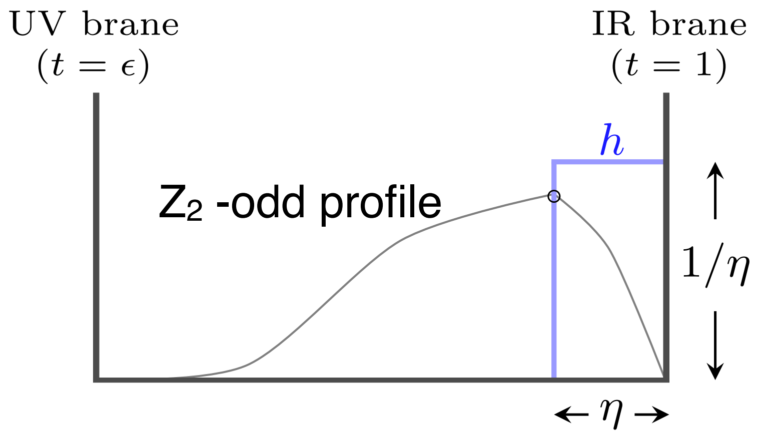
Loops in Higgs physics

Additional model dependence

- Yukawa interactions formally localised at IR brane: $S_{\rm IR} \ni \int d^4x \int_\epsilon^1 dt\, \sqrt{G}\, \delta(t-1) \left[\boldsymbol{Y}_d^{(5)}\, \bar{Q}\, \Phi\, d^c + \boldsymbol{Y}_u^{(5)} \bar{Q}\, i\sigma^2\, \Phi^*\, u^c + \text{h.c.} \right]$
  - electroweak symmetry breaking implies that the Z_2 -even and -odd quark profiles become discontinuous at the IR brane: $\boldsymbol{C}_n^{Q,q}(1^-) \neq \boldsymbol{C}_n^{Q,q}(1)$ and $\boldsymbol{S}_n^{Q,q}(1^-) \neq \boldsymbol{S}_n^{Q,q}(1) = 0$
  - 5D quark propagator is discontinuous at IR brane

- smear out (regularise) the Higgs profile, e.g. via a box-shaped function

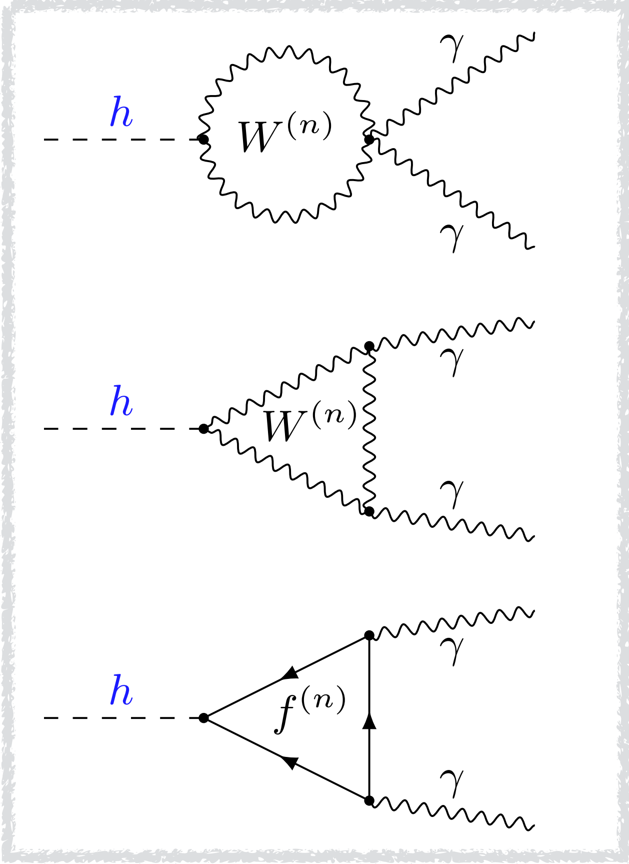
$$\delta(t-1) \to \delta^\eta(t-1) \equiv \begin{cases} \frac{1}{\eta} & , 1-\eta \leq t \leq 1 \\ 0 & , \text{else} \end{cases}$$



- new regulator η (box width) distinguishes between two models
  - brane-localized Higgs model: $\eta \ll \dfrac{v}{\Lambda_{\rm TeV}}$ with $\Lambda_{\rm TeV} = \epsilon M_{\rm Pl} \sim 10\, M_{\rm KK}$
  - narrow bulk-Higgs model: $\eta \gg \dfrac{v}{\Lambda_{\rm TeV}}$ and $\eta \ll \dfrac{v}{M_{\rm KK}}$

Higgs decay into two photons

- we have checked gauge invariance by calculating all diagrams including W-bosons, Goldstone scalars and ghost modes
- as in the SM it is allowed to calculate in unitary gauge
- amplitude parametrization: $\mathcal{A}(h \to \gamma\gamma) = C_{1\gamma}\,\frac{\alpha}{6\pi v}\,\langle\gamma\gamma|F_{\mu\nu}F^{\mu\nu}|0\rangle - C_{5\gamma}\,\frac{\alpha}{4\pi v}\,\langle\gamma\gamma|F_{\mu\nu}\tilde{F}^{\mu\nu}|0\rangle$



W-boson contribution (expanded in $m^2/M_{\rm KK}^2$, custodial RS model)

$$C_{1\gamma}^W \approx -\frac{21}{4}\left[\left(1-\frac{Lm_W^2}{M_{\rm KK}^2}\right)A_W(\tau_W)+\frac{Lm_W^2}{M_{\rm KK}^2}\right]$$

- $A_W(\tau_W) \approx 1.19$
- $L = kr\pi \approx 33.5$
- decoupling with KK scale

quark contribution (custodial RS model)

$$C_{1\gamma}^q \approx \left[1-\frac{2v^2}{3M_{\rm KK}^2}\mathrm{Re}\frac{(\boldsymbol{Y}_u\boldsymbol{Y}_u^\dagger\boldsymbol{Y}_u)_{33}}{(\boldsymbol{Y}_u)_{33}}\right]N_cQ_u^2\,A_q(\tau_t) \mp \frac{N_cQ_u^2\,v^2}{M_{\rm KK}^2}\mathrm{Tr}\,\boldsymbol{Y}_u\boldsymbol{Y}_u^\dagger \mp \frac{N_c(Q_u^2+Q_d^2+Q_\lambda^2)\,v^2}{M_{\rm KK}^2}\mathrm{Tr}\,\boldsymbol{Y}_d\boldsymbol{Y}_d^\dagger$$

$$C_{5\gamma}^q \approx -\frac{2v^2}{3M_{\rm KK}^2}\,\mathrm{Im}\left[\frac{(\boldsymbol{Y}_u\boldsymbol{Y}_u^\dagger\boldsymbol{Y}_u)_{33}}{(\boldsymbol{Y}_u)_{33}}\right]N_c\,Q_u^2\,B_q(\tau_t)$$

- $A_q(\tau_t) \approx 1.03,\ B_q(\tau_t) \approx 1.05$
- \- sign: brane Higgs scenario
- \+ sign: narrow-bulk Higgs scenario
- randomise Yukawa matrices (anarchy assumption)

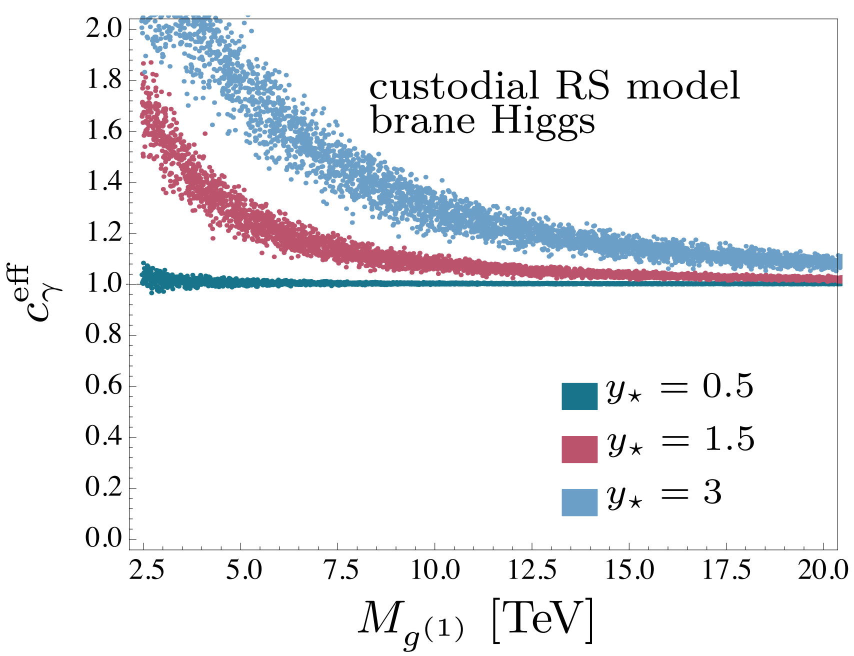
$$\left\langle \mathrm{Tr}\,\boldsymbol{Y}_q\boldsymbol{Y}_q^\dagger\right\rangle \approx N_g^2\,\frac{y_\star^2}{2} \quad \text{with} \quad |(\boldsymbol{Y}_q)_{ij}| \le y_\star$$

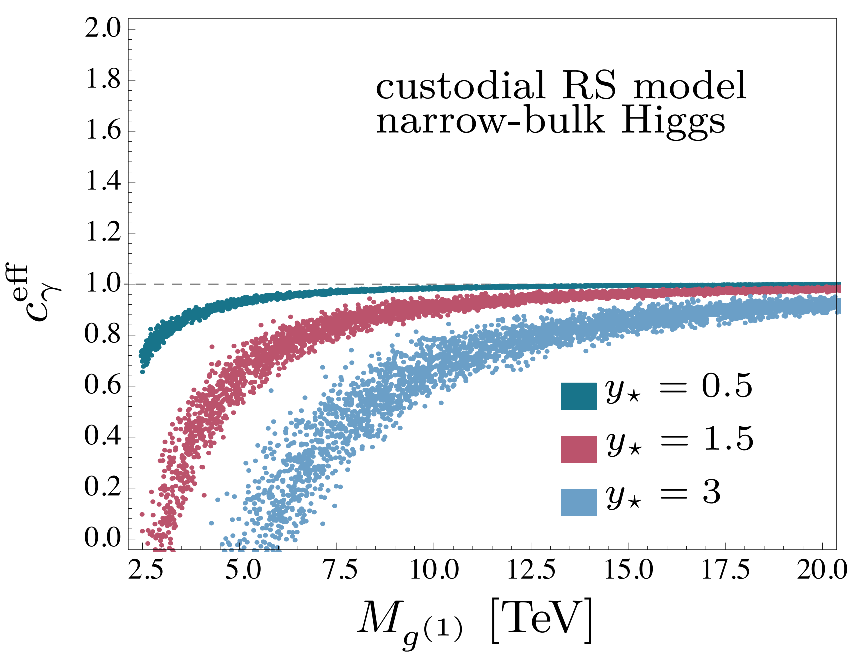
lepton contribution (custodial RS model)

- minimal embedding: $C_{1\gamma}^l \approx \mp Q_e^2\,\frac{v^2}{2M_{\rm KK}^2}\,\mathrm{Tr}\,\boldsymbol{Y}_e\boldsymbol{Y}_e^\dagger$

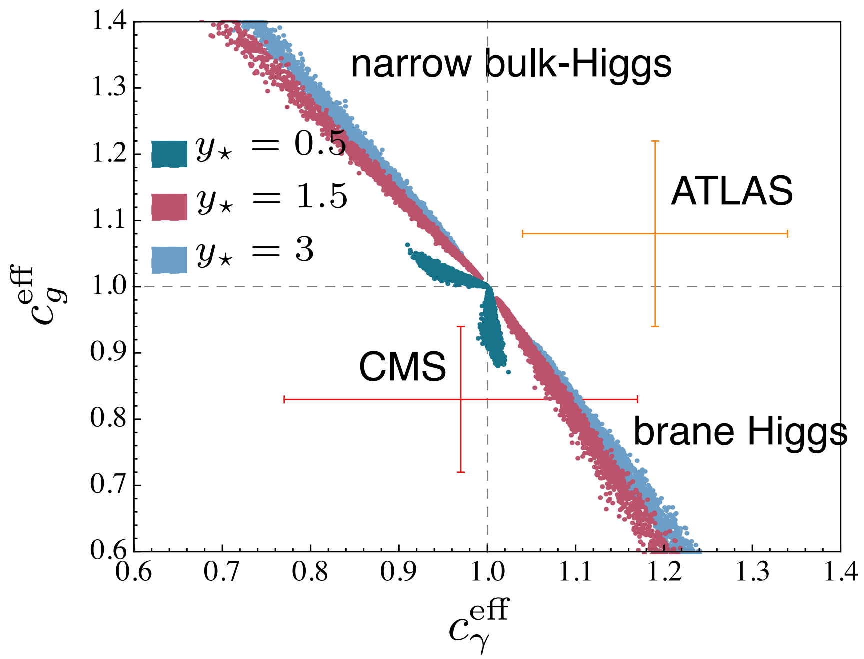
Loop-induced Higgs couplings

$h \to \gamma\gamma$ (custodial RS model)





$h \to \gamma\gamma$ vs. $h \to gg$



- $M_{g^{(1)}} = 2.45\, M_{\text{KK}}$
- $|(\boldsymbol{Y}_q)_{ij}| \le y_\star$

effective couplings

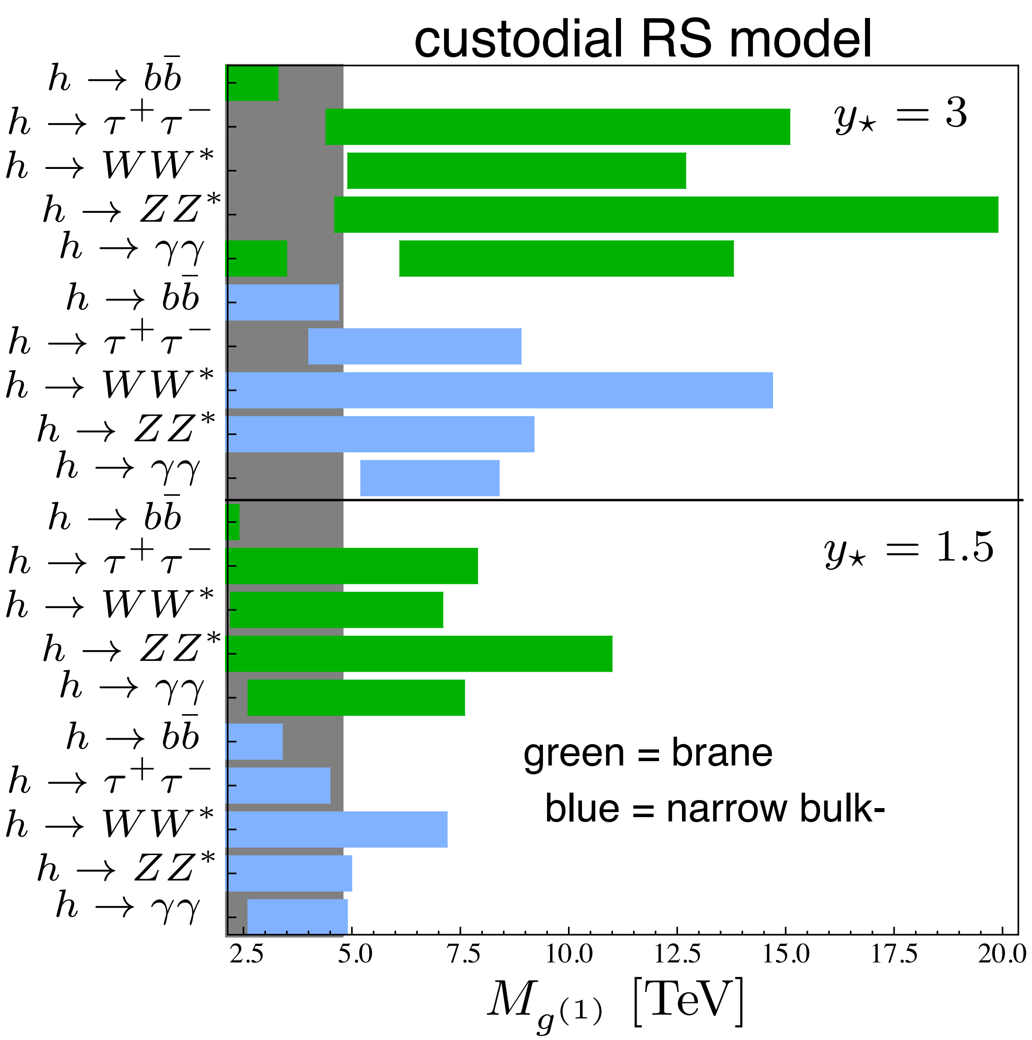
$$c_\gamma^{\text{eff}} = \frac{c_\gamma + N_c Q_u^2\, A_q(\tau_t)\, c_t - \frac{21}{4}\, A_W(\tau_W)\, c_W}{N_c Q_u^2\, A_q(\tau_t) - \frac{21}{4}\, A_W(\tau_W)} \approx 1 + \frac{v^2}{2M_{\text{KK}}^2}\left[(\pm 21.7 + 0.9)\, y_\star^2 - 5.1\right]$$

$$c_g^{\text{eff}} = \frac{c_g + A_q(\tau_t)\, c_t}{A_q(\tau_t)} \approx 1 + \frac{v^2}{2M_{\text{KK}}^2}\left[(\mp 36.0 - 3.3)\, y_\star^2 - 3.6\right]$$

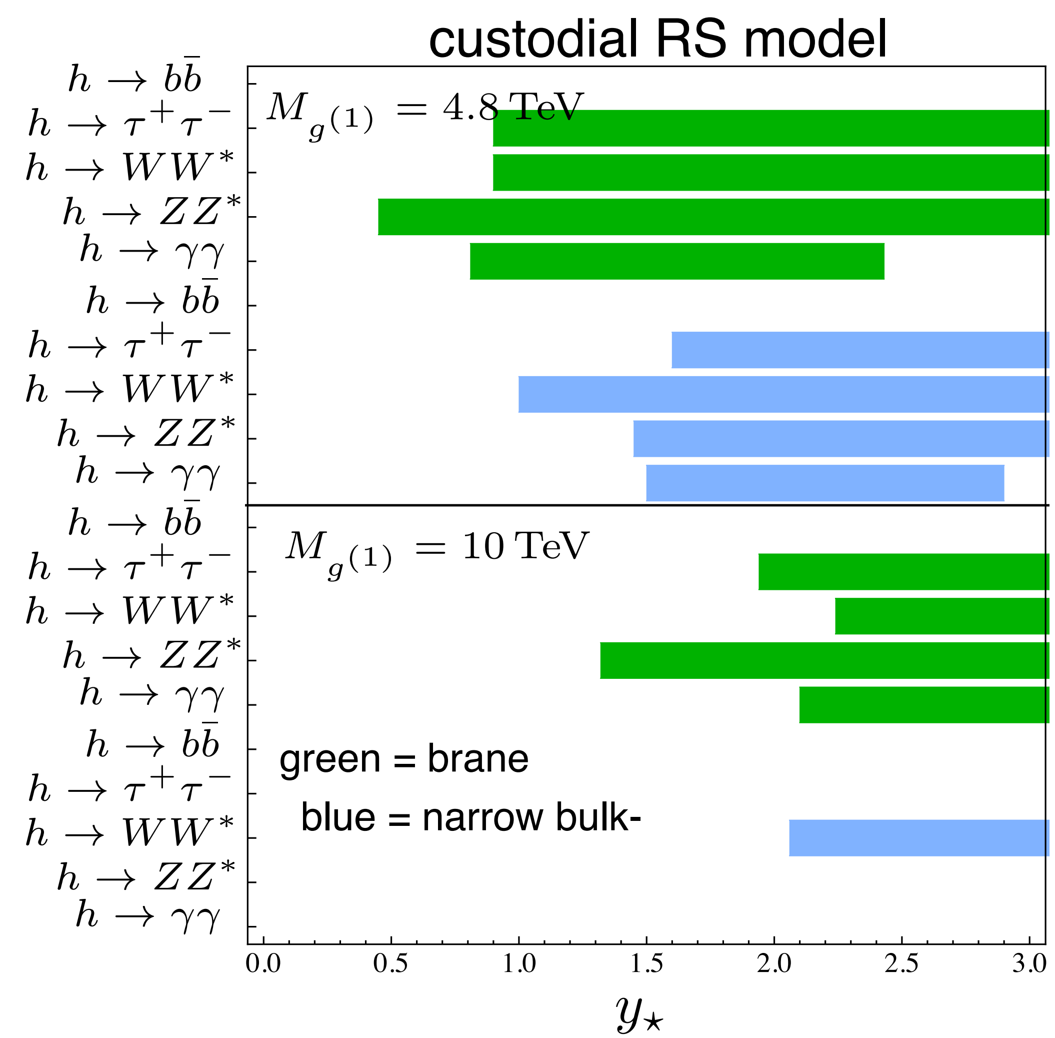
Compare signal rates with LHC data

- Signal rates: $$R_X \equiv \frac{(\sigma \cdot \mathrm{BR})(pp \to h \to X)_{\mathrm{RS}}}{(\sigma \cdot \mathrm{BR})(pp \to h \to X)_{\mathrm{SM}}} = \frac{\sigma(pp \to h)_{\mathrm{RS}}}{\sigma(pp \to h)_{\mathrm{SM}}} \, \frac{\Gamma(h \to X)_{\mathrm{RS}}}{\Gamma(h \to X)_{\mathrm{SM}}} \, \frac{\Gamma_h^{\mathrm{SM}}}{\Gamma_h^{\mathrm{RS}}}$$

bounds on KK gluon mass at 95% CL



bounds on maximal Yukawa value at 95% CL



- $|(\boldsymbol{Y}_q)_{ij}| \le y_\star$
- $M_{g^{(1)}} = 2.45\, M_{\mathrm{KK}}$

R_X	bb	$\tau\tau$	WW	ZZ	$\gamma\gamma$	
ATLAS	$0.2^{+0.7}_{-0.6}$	$1.4^{+0.5}_{-0.4}$	$1.00^{+0.32}_{-0.29}$	$1.44^{+0.40}_{-0.35}$	$1.57^{+0.33}_{-0.28}$	[ATLAS-CONF-2014-009]
CMS	$1.0^{+0.5}_{-0.5}$	$0.78^{+0.27}_{-0.27}$	$0.68^{+0.20}_{-0.20}$	$0.92^{+0.28}_{-0.28}$	$0.77^{+0.27}_{-0.27}$	[CMS-PAS-HIG-13-005]
average	$0.7^{+0.4}_{-0.4}$	$0.92^{+0.24}_{-0.22}$	$0.77^{+0.17}_{-0.16}$	$1.09^{+0.23}_{-0.22}$	$1.09^{+0.21}_{-0.19}$	

Conclusion

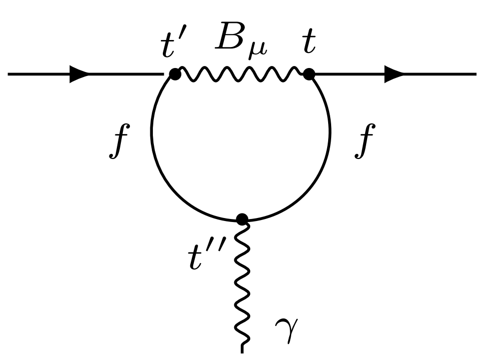
- 5D Calculation of the loop-induced Higgs couplings: $gg \to h,\, h \to \gamma\gamma$ with a distinction between the brane-localized and narrow-bulk Higgs scenario.

- Loop-induced Higgs couplings are very sensitive on the exchange of virtual fermionic KK excitations.

- Signal rates already give stringent bounds on the RS parameter space. These bounds are complementary and often stronger than those from electroweak precision observables and rare flavor-changing processes (custodial RS model).

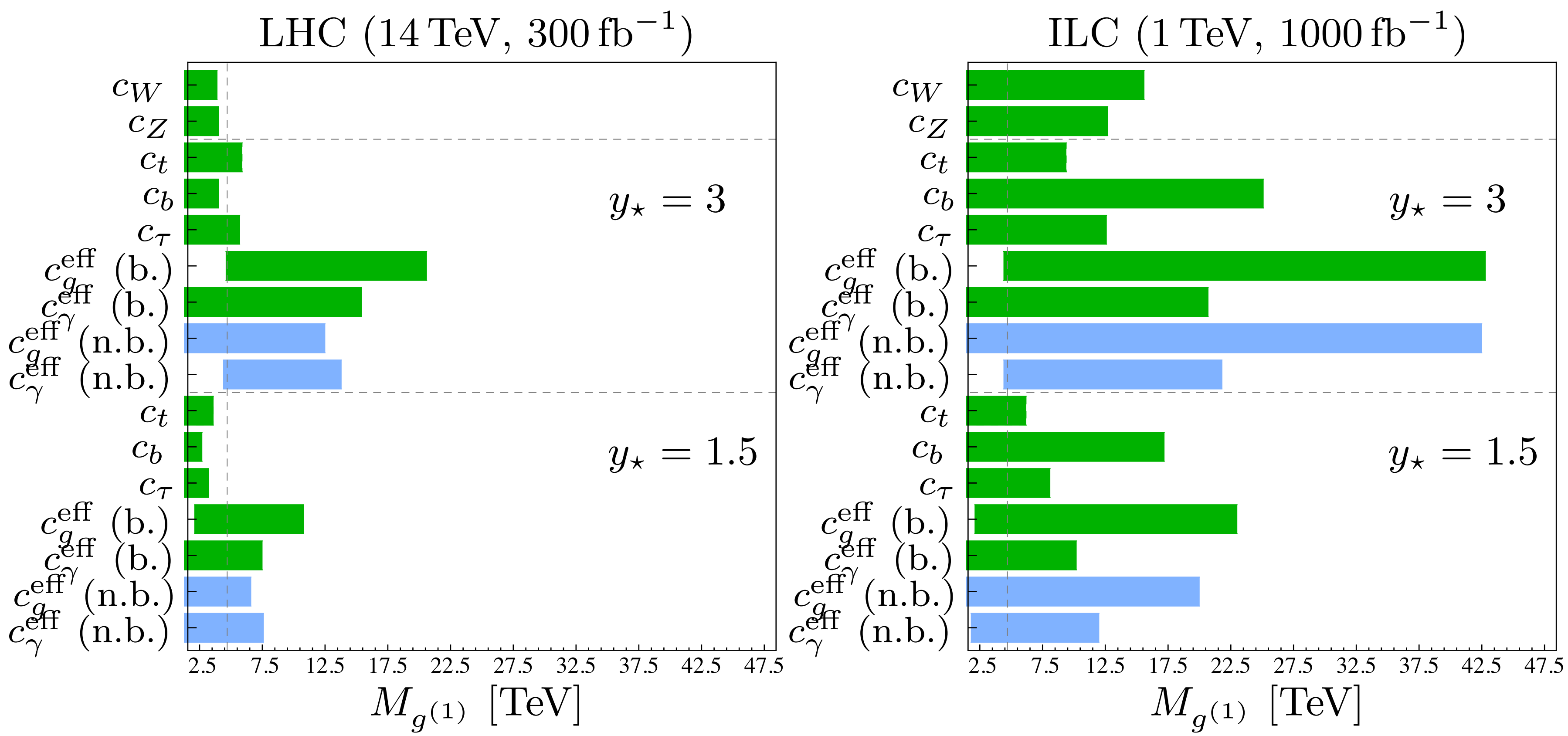
Current project

- 5D Calculation of magnetic dipole-operators: $b \to s\gamma,\, \mu \to e\gamma$
  - two independent KK sums
  - numerical integration of products of two 5D propagators



Higgs couplings: future sensitivities at LHC and ILC

bounds on $M_{g^{(1)}}$ at 95% CL (custodial RS model)



(n.b.) = narrow bulk-Higgs

(b.) = brane Higgs

$$|(\boldsymbol{Y}_q)_{ij}| \leq y_\star$$

$$M_{g^{(1)}} = 2.45\, M_{\rm KK}$$

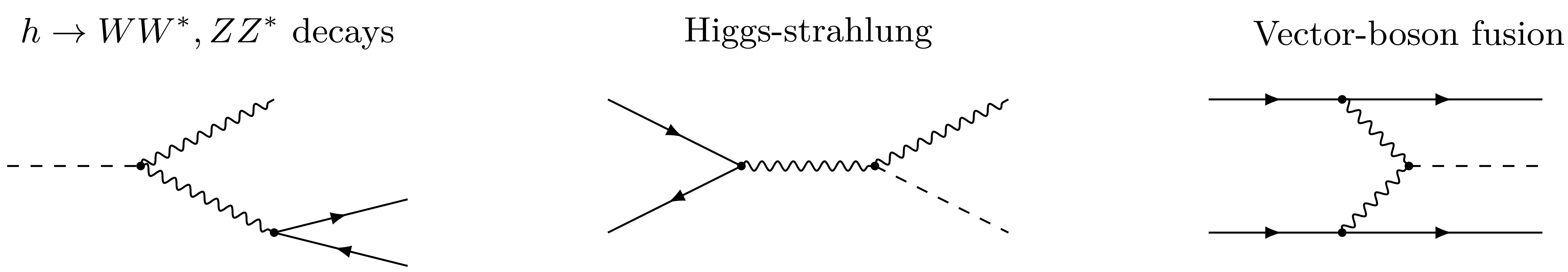
- LHC analysis:
  - brane Higgs: $M_{g^{(1)}} > 21\,\text{TeV} \times (y_\star/3)$
  - narrow bulk-Higgs: $M_{g^{(1)}} > 13\,\text{TeV} \times (y_\star/3)$
- ILC analysis:
  - brane and narrow bulk-Higgs: $M_{g^{(1)}} > 43\,\text{TeV} \times (y_\star/3)$

[Peskin:hep-ph/1207.2516]

- assume SM outcome
- constraint: $c_{W,Z} \leq 1$

$c_i - 1$	W	Z	t	b
LHC 14 TeV, 300 fb^{-1}	$(-0.069, 0)$	$(-0.077, 0)$	$(-0.154, 0.147)$	$(-0.231, 0.041)$
ILC 1 TeV, 1000 fb^{-1}	$(-0.004, 0)$	$(-0.006, 0)$	$(-0.044, 0.035)$	$(-0.003, 0.011)$
$c_i - 1$	τ	g	γ	
LHC 14 TeV, 300 fb^{-1}	$(-0.093, 0.132)$	$(-0.078, 0.10)$	$(-0.096, 0.059)$	
ILC 1 TeV, 1000 fb^{-1}	$(-0.013, 0.017)$	$(-0.014, 0.014)$	$(-0.032, 0.035)$	

Tree-level Higgs production and decay



Example: $h \to WW^* \to W \bar{f}_1 f_1' \to \bar{f}_2 f_2' \bar{f}_1 f_1'$

[RM,Neubert,Schmell,hep-ph/1408.4456]

- hWW coupling: $c_W \approx 1 - \frac{m_W^2}{2M_{\rm KK}^2}\left(\frac{3}{2}L - 1 + \frac{1}{2L}\right)$
  - $L = kr\pi \approx 33.5$
- off-shell 5D W-boson propagator: $\sum_{n=0}^{\infty} \frac{\chi_W^n(1)\chi_W^n(\epsilon)}{m_{W_n}^2 - p^2} \approx \frac{\chi_W(1)\chi_W(\epsilon)}{m_W^2 - p^2} - \frac{1}{2M_{\rm KK}^2}\left(1 - \frac{1}{L}\right)$
- modification of the $W\bar{f}f'$ coupling: $c_{\Gamma_W} = \frac{\Gamma(W \to \bar{f}f')_{\rm RS}}{\Gamma(W \to \bar{f}f')_{\rm SM}} \approx 1 - \frac{m_W^2}{2M_{\rm KK}^2}\frac{1}{2L}$

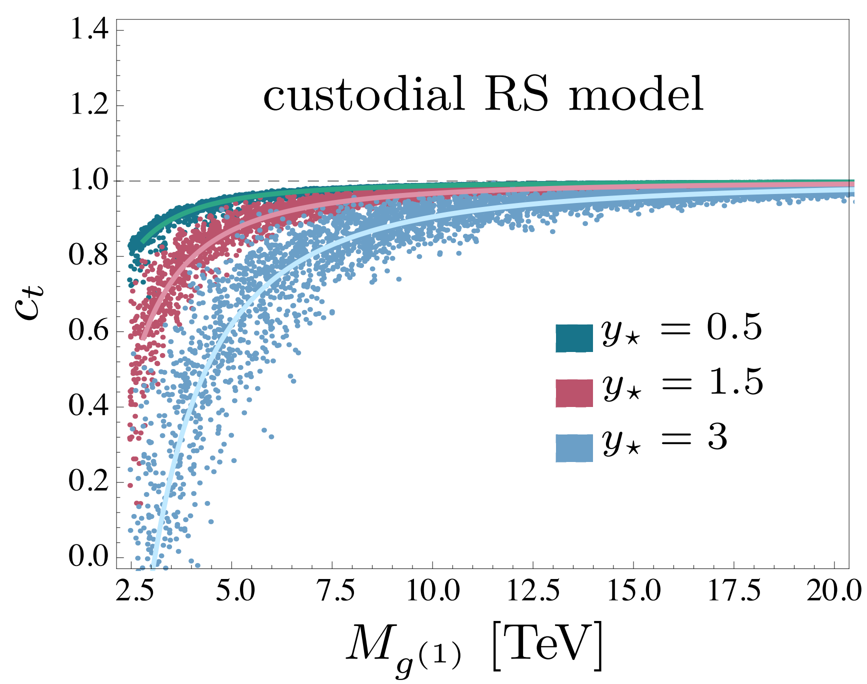
⇒ To good approximation the main effects can be accounted for by a multiplicative rescaling of the SM decay rates and production cross sections

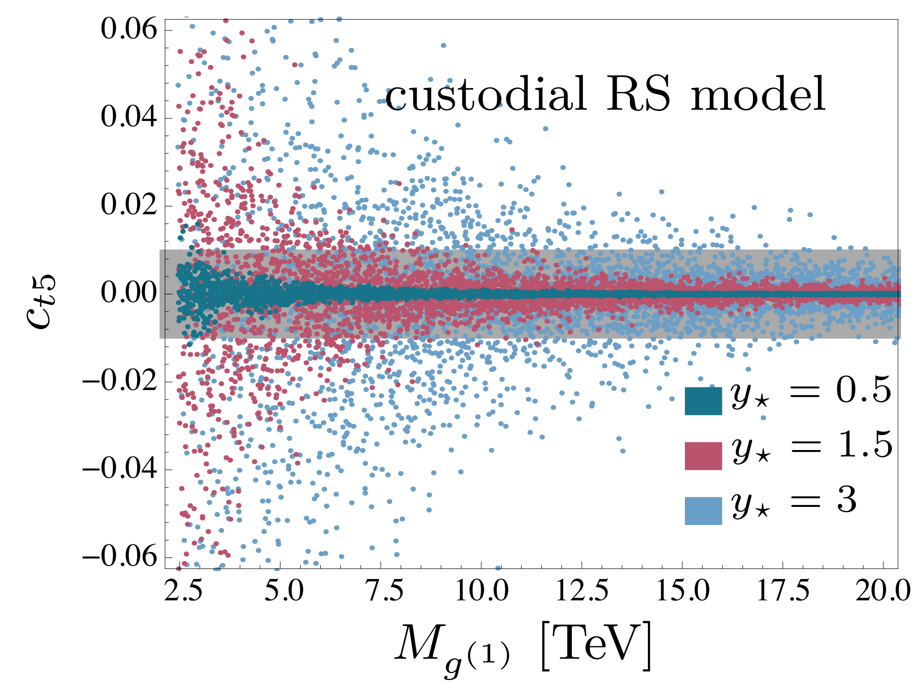
Higgs couplings: tree-level

hVV coupling (custodial RS model): $$c_W \approx c_Z \approx 1 - 0.078 \left(\frac{5\,\text{TeV}}{M_{g^{(1)}}}\right)^2$$

- directly sensitive on KK gluon mass

$h\bar{t}t$ coupling (custodial RS model) $$c_f + i c_{f5} = 1 - \epsilon_f - \frac{L m_W^2}{4 M_{\text{KK}}^2} - \frac{v^2}{3 M_{\text{KK}}^2} \frac{(\boldsymbol{Y}_f \boldsymbol{Y}_f^\dagger \boldsymbol{Y}_f)_{33}}{(\boldsymbol{Y}_f)_{33}} + \ldots$$





- 5000 scatter points
- $|(\boldsymbol{Y}_q)_{ij}| \leq y_\star$

- electron EDM (at 90 % CL): $d_e < 8.7 \cdot 10^{-29} e\, cm \quad \rightarrow \quad c_{t5} \leq 0.01$ [Brod,Haisch,Zupan,hep-ph/1310.1385]

Higgs couplings

Effective Lagrangian in the broken Higgs phase at the electroweak scale

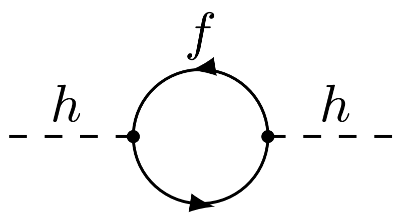
$$\mathcal{L}_{\rm eff} = c_W \frac{2m_W^2}{v_{\rm SM}} hW_\mu^+ W^{-\mu} + c_Z \frac{m_Z^2}{v_{\rm SM}} hZ_\mu Z^\mu - \sum_{f=t,b,\tau} \frac{m_f}{v_{\rm SM}} h\bar{f}\,(c_f + c_{f5}\, i\gamma_5)\, f$$
$$+ c_g \frac{\alpha_s}{12\pi v_{\rm SM}} hG_{\mu\nu}^a G^{a,\mu\nu} - c_{g5} \frac{\alpha_s}{8\pi v_{\rm SM}} hG_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + c_\gamma \frac{\alpha}{6\pi v_{\rm SM}} hF_{\mu\nu} F^{\mu\nu} - c_{\gamma 5} \frac{\alpha}{4\pi v_{\rm SM}} hF_{\mu\nu} \tilde{F}^{\mu\nu} + \dots .$$

- SM: $c_W = c_Z = c_f = 1$ and $c_{f5} = c_g = c_{g5} = c_\gamma = c_{\gamma 5} = 0.$
- not complete list of operators; but remaining ones are subdominant, e.g. $hZ_\mu \bar{f}\gamma^\mu f\,,\, hZ_\mu \bar{f}\gamma^\mu \gamma_5 f$

Motivation - hierarchy puzzles of the SM

Gauge Hierarchy Puzzle

- why is the Higgs so light, $m_h^2 \ll M_{\rm Pl}^2$(roughly 32 orders of magnitude) ?
- Higgs mass operator not protected by any symmetry (radiatively unstable)



$$\Rightarrow \qquad \delta m_h^2 = \frac{\mathcal{O}(1)}{16\pi^2} \times \left(\Lambda_{\rm UV}^2 + m_f^2 \log(\Lambda_{\rm UV}/m_f) + \ldots\right)$$

Flavour Hierarchy Puzzle

- why do Yukawa matrices have a hierarchical pattern (flavour puzzle) ?

$$|Y_u| \sim \begin{pmatrix} 4.3\,10^{-6} & 4.8\,10^{-4} & 8.6\,10^{-3} \\ 2.8\,10^{-5} & 2.8\,10^{-3} & 6.4\,10^{-2} \\ 2.7\,10^{-4} & 3.3\,10^{-2} & 0.8 \end{pmatrix} \qquad \Leftrightarrow \qquad y_t \sim 1,\; y_c \sim 10^{-3},\; y_u \sim 10^{-6}$$

- new physics at the TeV scale should explain the suppression of FCNC processes (GIM-like mechanism)

$$\mathcal{L}_{\rm eff} = \sum_i \frac{c_i}{\Lambda^2}\,\mathcal{O}_i \qquad \overset{c_i \sim \mathcal{O}(1)}{\longleftrightarrow}$$

observable	$\mathcal{O}_i$	Λ [TeV]
ϵ_K	$(ds^c)(ds^c)$	$10^4 - 10^5$
Δm_K	$(ds^c)(ds^c)$	$10^3 - 10^3$
Δm_D	$(cu^c)(cu^c)$	$10^2 - 10^3$
Δm_{B_d}	$(bd^c)(bd^c)$	$10^2 - 10^3$