5D Perspective on Higgs Production via Gluon Fusion at the Boundary of a Warped Extra Dimension (I)

in collaboration with K. Novotny, C. Schmell and M. Neubert
based on [1303.5702 hep-ph]

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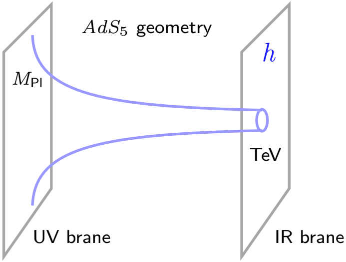
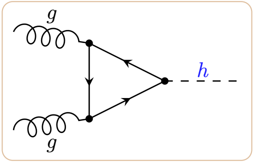
Introduction

With the discovery of a Higgs-like boson, the gauge hierarchy problem - the question about the ultra-violet sensitivity of the scalar sector and the stability of the Higgs potential under quantum fluctuations - is more pressing than ever.

One attractive approach enlarges space-time by a slice of AdS_5, cut off by two 4D branes



[Randall & Sundrum '99]



gluon fusion is the main Higgs production process at the LHC

Higgs physics can be used to probe warped extra dimensional theories - especially gluon fusion $gg \to h$ (as well as $h \to \gamma\gamma$) can receive large effects - in addition to electroweak precision measurements.

Outline

- Brief introduction to the minimal Randall-Sundrum model with a brane-localized Higgs sector

- Discussion of the gluon-fusion process $gg \to h$ with a focus on the reliability of the result, using the 5D perspective

- Summary

Part II: Results in more detail, extension to the custodial model, phenomenology will be covered by C. Schmell

The Randall-Sundrum model (minimal version)

[Randall & Sundrum '99]

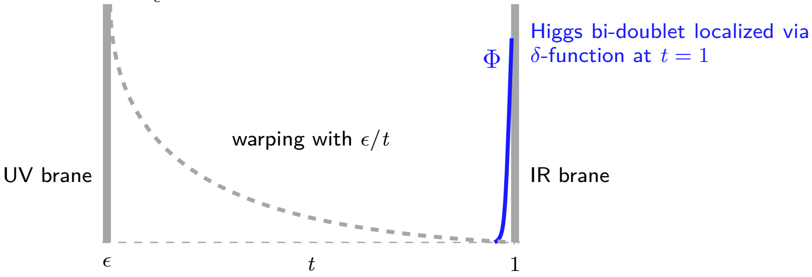
- Idea: only one scale, which is the Planck mass $M_{\text{Pl}} \sim (M_{\text{Pl}(5)}, k, 1/r)$
- Extra dimension is an orbifold S^1/Z_2 (allows for chiral fermions in 5D) in a slice of AdS_5, labeled by $t \in [\epsilon, 1]$ ($t = \epsilon e^{kr|\phi|}$)
- Metric: $$ds^2 = \frac{\epsilon^2}{t^2}\left(dx^\mu \, dx_\mu - M_{\text{KK}}^{-2} \, dt^2\right)$$

Warping strength

$$\epsilon = e^{-kr\pi} \stackrel{!}{=} \frac{M_{\text{EW}}}{M_{\text{Pl}}} \approx 10^{-16}$$

- Solve the gauge hierarchy problem via a moderate tuning $kr \sim 12$, which can be stabilized by a bulk scalar [Goldberger & Wise '99]

$$\mathcal{L}_{\text{Higgs}} = \underbrace{\sqrt{|G_{\text{IR}}|}}_{\epsilon^4} \lambda_5 \left(\Phi^\dagger \Phi - v_5^2/2\right)^2 \quad \longrightarrow \quad \lambda_5 \left(\Phi^\dagger \Phi - (\epsilon v_5)^2/2\right)^2$$



The Randall-Sundrum model (minimal version)

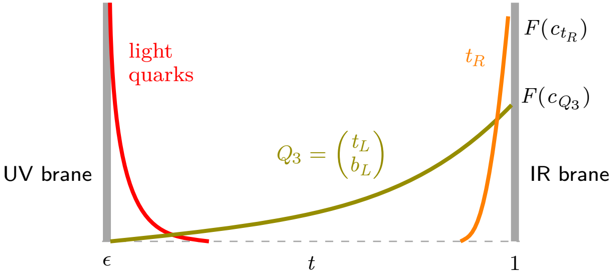
- Bulk gauge group as in SM: $SU(3)_c \times SU(2)_L \times U(1)_Y$
- Particle content: SM-like particles + heavy resonances (infinite sum of KK particles)
- New particles have masses $m_n \sim n\,\pi M_{\mathsf{KK}}$, where $M_{\mathsf{KK}} = k\epsilon \sim$ few TeV
- Explain flavor puzzle by geometrical localization along the extradimension with anarchic bulk mass parameters $c_{Q_i,q_i} \sim \mathcal{O}(1)$ and 5D Yukawas $Y_{q_{ij}} \sim \mathcal{O}(1)$
  [Grossmann & Neubert '99, Gherghetta & Pomarol '00, Huber & Shafi '00]

Effective Yukawa ($q = u, d$)

$$\boldsymbol{Y}_q^{\text{eff}} \sim F(\boldsymbol{c}_Q)\,\boldsymbol{Y}_q F(\boldsymbol{c}_q)$$

Overlap with Higgs (IR brane)

$$F(c) = \sqrt{\frac{1+2c}{1-\epsilon^{1+2c}}}$$

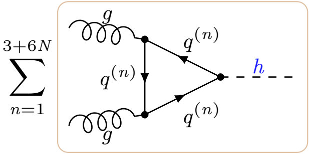


4D perspective on Higgs production via $gg \to h$

How large is the KK contribution for a brane Higgs ?

Result insensitive to KK modes above the cutoff ?

In AdS_5, the 4D momentum cutoff is position dependent $\Lambda(t) = \epsilon M_{\text{Pl}}/t$.

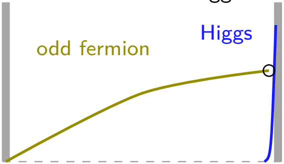


[hep-th/1005.4315]

- $\Lambda(1) = \epsilon M_{\text{Pl}} \approx 10\, M_{\text{KK}}$ (only a few physical modes)
- Orbifold odd fermion profiles $S_n(t)$ require a regularization of the δ-function: new scale η
- KK contribution in the brane Higgs scenario:

$$\Delta C_1 \approx -\frac{v^2}{2M_{\text{KK}}^2} \text{Tr}\left[\boldsymbol{Y}_q \boldsymbol{Y}_q^\dagger\right] \quad \text{(negative correction to SM)}$$

brane-localized Higgs

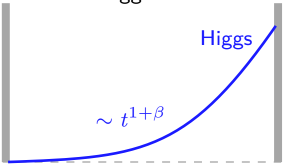


[hep-th/1006.5939]

- Bulk Higgs result: $\Delta C_1 > 0$ (positive correction to SM)
- Narrow bulk Higgs: moving the Higgs profile towards the IR brane ($\beta \to \infty$), one can extrapolate the above result
- KK contribution in the narrow bulk-Higgs scenario:

$$\Delta C_1 \approx +\frac{v^2}{2M_{\text{KK}}^2} \text{Tr}\left[\boldsymbol{Y}_q \boldsymbol{Y}_q^\dagger\right] \quad \text{(positive correction to SM)}$$

bulk-Higgs scenario



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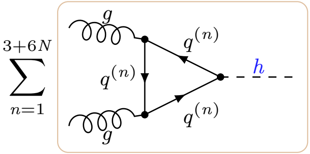

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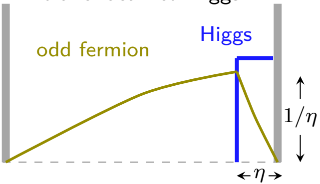

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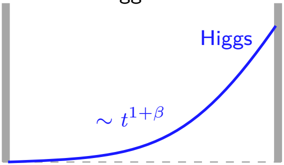

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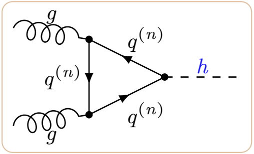

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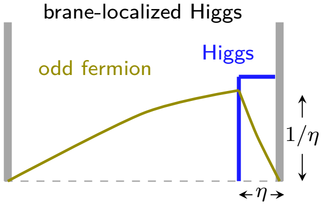

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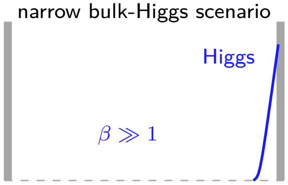

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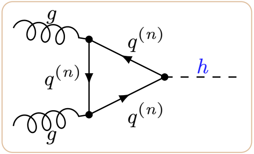

How can we understand the two distinct results ?

$$\text{KK contribution} \sim \sum_{n=4}^{3+6N} \frac{v\, g^h_{nn}(\eta)}{m_n}$$

[hep-th/1204.0008]

The sum is conditionally convergent.

Heavy modes with masses $m_n \sim v|Y_q|/\eta$ can probe the Higgs profile and contribute significantly to the sum.

Excluding them, we obtain the brane-localized result.

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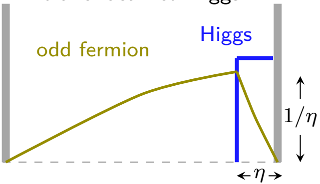
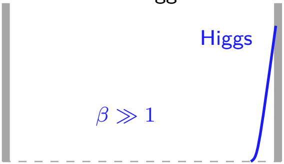
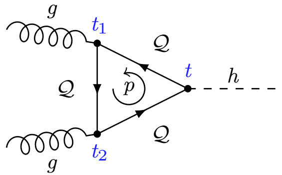
brane-localized Higgs



narrow bulk-Higgs scenario





- Idea: use the 5D fermion propagator for three generations and with full momentum dependence
- Advantage: the sum over all KK modes is implicit using the correct ordering of terms
- For the 4D momentum integral, we use dimensional regularization with $d = 4 - 2\hat{\epsilon}$

$$\mathcal{A}(gg \to h) = i\frac{4\pi\alpha_s}{\sqrt{2}}\delta_{ab}\int \frac{d^d p}{(2\pi)^d}\int_\epsilon^1 dt_1 \int_\epsilon^1 dt_2 \int_\epsilon^1 dt\, \delta_h^\eta(t-1) \sum_{q=u,d}$$

$$\mathrm{Tr}\left[\begin{pmatrix} 0 & \boldsymbol{Y}_q \\ \boldsymbol{Y}_q^\dagger & 0 \end{pmatrix} \boldsymbol{S}_q(t, t_2; p - k_2) \not{\epsilon}(k_2) \boldsymbol{S}_q(t_2, t_1; p) \not{\epsilon}(k_1) \boldsymbol{S}_q(t_1, t; p + k_1)\right]$$

Nontrivial momentum integral can be expressed by ($\overline{\text{MS}}$ scheme, $p_E^2 = -p^2$)

$$I_+(m^2) \equiv -\frac{\mu^{2\hat{\epsilon}} e^{\hat{\epsilon}\gamma_E}}{\Gamma(1-\hat{\epsilon})}\int_0^\infty dp_E\, p_E^{-2\hat{\epsilon}} \frac{\partial}{\partial p_E} T_+(p_E^2 - m^2 - i0)$$

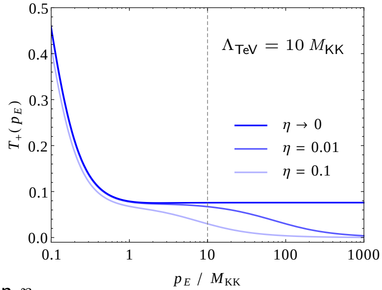
$T_+(p_E^2)$ is an analytic function depending on the RL/LR components of the 5D propagator $\boldsymbol{S}_q(t, t'; p)$ and the regularization scale η.

High momentum behavior of the function $T_+(p_E^2)$

Two characteristic regions

$$(1) \quad M_{\text{KK}} \ll p_E \ll \frac{v|Y_q|}{\eta}$$

$$(2) \quad p_E \gg \frac{v|Y_q|}{\eta}$$



High momentum behavior of T_+ depends on η

$$\underset{(1)}{\eta_{\text{brane-localized Higgs}} \ll \frac{v|Y_q|}{\Lambda_{\text{TeV}}}} \ll \underset{(2)}{\eta_{\text{narrow bulk-Higgs}} \ll \frac{v|Y_q|}{M_{\text{KK}}}}$$

As in the 4D picture one can distinguish two scenarios, where

(1) η is treated as an unphysical parameter, i.e. physical KK particles can not resolve the Higgs profile (brane-localized Higgs scenario).

(2) η represents the physical width of the Higgs profile, i.e. physical KK particles near the cutoff can resolve the Higgs profile. (narrow bulk-Higgs scenario)

Let's have a closer look at the loop-momentum integral $I_+(m^2)$

Loop-momentum integral given by (using dimensional regularization)

$$I_+(m^2) = -\frac{\mu^{2\hat{\epsilon}}\, e^{\hat{\epsilon}\gamma_E}}{\Gamma(1-\hat{\epsilon})} \int_0^\infty dp_E\, p_E^{-2\hat{\epsilon}} \frac{\partial}{\partial p_E} T_+(p_E^2 - m^2 - i0)\,.$$

Study toy model, that captures all important features of the exact result
$(t_{0,1,3} \equiv t_{0,1,3}(\boldsymbol{Y}_q),\ t_2 \equiv t_2(\boldsymbol{c}_{Q,q}, \boldsymbol{Y}_q))$

$$T_+^{\text{model}}(p_E^2) = \frac{t_0 - t_1 - t_2}{1 + \hat{p}_E^2} + \frac{t_2}{\sqrt{1 + \hat{p}_E^2}} + \frac{t_3}{\sqrt{(t_3/t_1)^2 + (\eta \hat{p}_E)^2}}\,.$$

Findings:

Keep regulator $\hat{\epsilon}$

- Limit $\eta \to 0$ commutes with momentum integration, leading to the well-defined result $I_+^{\text{model}}(0) = t_0 - t_1$ $(\Delta C_1 < 0)$. The regulator $\hat{\epsilon}$ has the additional effect of regularizing the 5D propagator. (brane Higgs scenario)

Remove regulator $\hat{\epsilon}$ at finite $\eta \neq 0$

- Performing the momentum integration yields $I_+^{\text{model}}(0) = t_0$ $(\Delta C_1 > 0)$, with a result independent of the Higgs shape η. (narrow bulk-Higgs scenario)

Power corrections

Review $I_+(p_E^2)$ using a hard momentum regularization with cut-off $\Lambda(1) = \epsilon M_{\text{Pl}} = \Lambda_{\text{TeV}}$

$$I_+^{\text{model}}(0) = t_0 - t_1 - \frac{3t_2}{2}\frac{M_{\text{KK}}}{\Lambda_{\text{TeV}}} \qquad \text{(brane-localized Higgs)}$$

$$I_+^{\text{model}}(0) = t_0 - \frac{3t_3}{2}\frac{M_{\text{KK}}}{\eta\Lambda_{\text{TeV}}} \qquad \text{(narrow bulk-Higgs)}$$

Threshold correction are enhanced by $1/\eta$ in the narrow bulk-Higgs scenario. For small $\eta \lesssim M_{\text{KK}}/\Lambda_{\text{TeV}}$, theory is not under perturbative control anymore (transition region).

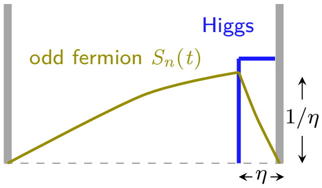
One can relate the power corrections to higher dimensional operators that involve a fermion bilinear containing a single derivative $M_{\text{Pl}}^{-1} E_a^A iD_A \gamma^a = \Lambda^{-1}(t)(i\partial\!\!\!/ - \gamma_5 M_{\text{KK}} \partial_t)$.

brane-localized Higgs

$$\partial_t\, S_n(t)\Big|_{1^-} \sim \mathcal{O}(1) \quad \to \quad \left(\frac{M_{\text{KK}}}{\Lambda_{\text{TeV}}}\right)^n$$

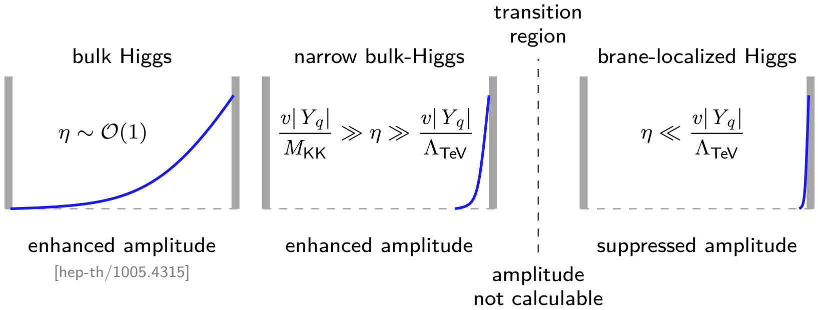
narrow bulk-Higgs

$$\partial_t\, S_n(t)\Big|_{1^-} \sim \frac{\mathcal{O}(1)}{\eta} \quad \to \quad \left(\frac{M_{\text{KK}}}{\eta\Lambda_{\text{TeV}}}\right)^n$$



Summary

- Randall-Sundrum models provide attractive scenarios that can address the gauge hierarchy problem and the flavor puzzle
- Higgs physics can be used as a probe for warped extra dimensional models
- Result of gluon fusion for the brane-localized Higgs scenario is calculable (finite) and UV safe (higher KK modes decouple)



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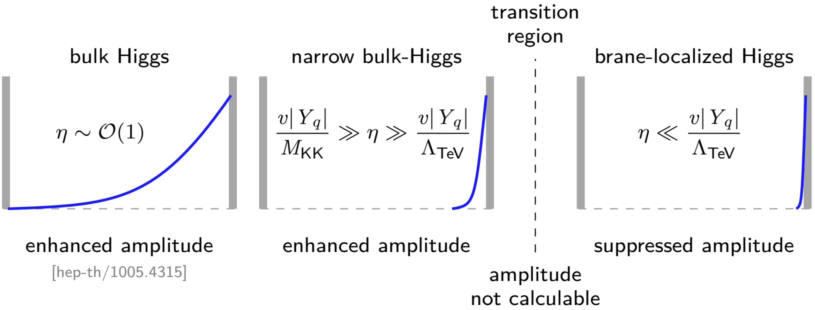


many thanks for your attention !

Relevance of higher-dimensional operator $|\Phi|^2(G^a_{\mu\nu})^2$?

Consider a brane-localized Higgs sector and add the action

$$S_{\text{eff}} = \int d^4x \int_\epsilon^1 \frac{dt}{t}\, c_{\text{eff}}\, \delta(t-1)\, \frac{\Phi^\dagger \Phi}{\Lambda^2_{\text{TeV}}} \frac{g^2_{s,5}}{4} \mathcal{G}^a_{\mu\nu}\mathcal{G}^{\mu\nu,a},$$

which changes the KK contribution for gluon fusion to

$$\Delta C_1 \approx -N_g^2 |Y_*|^2 \frac{v^2}{M^2_{\text{KK}}} + c_{\text{eff}}\, \frac{3}{4} \left(\frac{4\pi v}{\Lambda_{\text{TeV}}}\right)^2,$$

where N_g is the number of generations and $Y_\star$ the typical size of a Yukawa matrix element. Contributions from higher dimensional operators are negligible for

$$c_{\text{eff}} \ll \frac{4}{3} \frac{\Lambda^2_{\text{TeV}}}{M^2_{\text{KK}}} \frac{1}{16\pi^2} N_g^2 |Y_\star|^2 \approx 7.6 |Y_\star|^2 \quad \text{for} \quad \Lambda_{\text{TeV}} \approx 10\, M_{\text{KK}}.$$

Answer: For $Y_\star \sim \mathcal{O}(1)$ and $c_{\text{eff}} \sim 1$ (conservative choice), higher-dimensional operators are numerically subleading.

No parametric suppression in the brane-localized scenario ?

Under the assumptions (worst case) that

(1) the 5D Yukawas saturate the perturbativity bound, i.e. $y_\star \sim M_{\mathrm{KK}}/\Lambda_{\mathrm{TeV}}$,

(2) $c_{\mathrm{eff}} \sim \mathcal{O}(1)$ and independet of Yukawa couplings,

then there is no parametric suppression of the higher dimensional operator contribution relative to the loop-induced KK contributions.

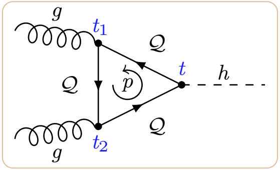
But, both assumptions above can not be taken for granted:

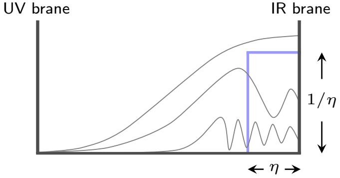
(1) Natural Yukawas are always $\mathcal{O}(1)$ parameters, i.e. the loop contribution is unsuppressed when increasing the cutoff Λ_{TeV}. The only physical effect is that the theory arrives at a strong coupling regime in the Yukawa sector.

(2) It is not unlikely that two gluons and the Higgs are coupled together by a quark condensate, involving some powers of Yukawas. In this case, higher-dimensional operators would be at least suppressed by $(M_{\mathrm{KK}}/\Lambda_{\mathrm{TeV}})^3$, one power higher than the loop contribution.

Even if we accept the assumptions, the conclusion would be that we have to add the operator contribution to the loop contribution and take the numerically small effect into account. Still, our result is not UV sensitive in the sense that the effective field theory is under control (compare to chiral perturbation theory).

Detailed classification of models





Model	bulk Higgs	narrow bulk-Higgs	transition region	brane Higgs
Higgs profile width	$\eta = \mathcal{O}(1)$	$\frac{v\lvert Y_q\rvert}{\Lambda_{\text{TeV}}} \ll \eta \ll \frac{v\lvert Y_q\rvert}{M_{\text{KK}}}$	$\eta \sim \frac{v\lvert Y_q\rvert}{\Lambda_{\text{TeV}}}$	$\eta \ll \frac{v\lvert Y_q\rvert}{\Lambda_{\text{TeV}}}$
Power corrections	$\sim \frac{M_{\text{KK}}}{\Lambda_{\text{TeV}}}$	$\sim \frac{M_{\text{KK}}}{\eta\Lambda_{\text{TeV}}}$	$\sim \frac{M_{\text{KK}}}{v\lvert Y_q\rvert}$	$\sim \frac{M_{\text{KK}}}{\Lambda_{\text{TeV}}}$
Higgs profile	resolved by all modes	resolved by high-momentum modes	partially resolved by high-momentum modes	not resolved
$\mathcal{A}(gg \to h)$	enhanced [hep-ph/1006.5939]	enhanced	not calculable	suppressed

Derivation of the 5D quark propagator

5D Dirac equation given by

$$\left(\not{p} - M_{\mathsf{KK}}\gamma_5 \frac{\partial}{\partial t} - M_{\mathsf{KK}}\,\mathcal{M}_q(t)\right) \boldsymbol{S}^q(t,t';p) = \delta(t-t')$$

with

$$\mathcal{M}_q(t) = \frac{1}{t}\begin{pmatrix} \boldsymbol{c}_Q & 0 \\ 0 & -\boldsymbol{c}_q \end{pmatrix} + \frac{v}{\sqrt{2}M_{\mathsf{KK}}}\delta_v^\eta(t-1)\begin{pmatrix} 0 & \boldsymbol{Y}_q \\ \boldsymbol{Y}_q^\dagger & 0 \end{pmatrix}$$

use $\boldsymbol{S}^q(t,t';p) = \left[\boldsymbol{\Delta}_{LL}^q(t,t';-p^2)\not{p}P_R + \boldsymbol{\Delta}_{RL}^q(t,t';-p^2)P_R\right] + P_R \leftrightarrow P_L$

- coupled system of 2 differential equations with 16 coefficients
- jump conditions at $t = t'$ reduce to 8 coefficients
- boundary conditions in the UV and IR each reduce 4 coefficients
- unique solutions in bulk $t \in [\epsilon, 1-\eta]$ and sliver region $t \in [1-\eta, 1]$

Another approach for $\eta \to 0$: modified boundary conditions at the IR brane

$$\begin{pmatrix} \frac{v}{\sqrt{2}M_{\mathsf{KK}}}\tilde{\boldsymbol{Y}}_q^\dagger & 1 \end{pmatrix} \boldsymbol{\Delta}_{LL}^q(1^-,t';-p^2) = \begin{pmatrix} 1 & -\frac{v}{\sqrt{2}M_{\mathsf{KK}}}\tilde{\boldsymbol{Y}}_q \end{pmatrix} \boldsymbol{\Delta}_{RL}^q(1^-,t';-p^2) = 0$$

Details on the propagator function $T_+(p_E^2)$

The $gg \to h$ amplitude involves three propagators $\boldsymbol{S}_q$, that can be simplified (using orthogonality relations when integrating over t_1 and t_2) such that we only need to know the RL/LR components coupling singlet with doublet quarks,

$$T_+(p_E^2) = \sum_{q=u,d} \frac{-v}{\sqrt{2}} \int_\epsilon^1 dt\, \delta_h^\eta(t-1)\, Tr\left[\begin{pmatrix} 0 & \boldsymbol{Y}_q \\ \boldsymbol{Y}_q^\dagger & 0 \end{pmatrix} \frac{\boldsymbol{\Delta}_{RL}^q(t,t;p_E^2) + \boldsymbol{\Delta}_{LR}^q(t,t;p_E^2)}{2} \right] .$$

Inserting the exact solutions for $\boldsymbol{\Delta}_{RL}$ in the sliver region, we find the result

$$T_+(p_E^2) = \sum_{q=u,d} \int_\epsilon^1 dt\, \delta_h^\eta(t-1)\, Tr\left\{ \frac{\boldsymbol{X}_q^2}{\boldsymbol{S}_q \sinh 2\boldsymbol{S}_q} \right.$$
$$\left. \times \left[\sinh^2 \boldsymbol{S}_q + \mathcal{C}^2(t)\, \boldsymbol{Z}_q^{\eta,1}(p_E^2)\, \frac{1}{\boldsymbol{N}_q^{\eta,1}(p_E^2)} - \mathcal{S}^2(t)\, \frac{\boldsymbol{N}_q^{\eta,2}(p_E^2) - 1}{\boldsymbol{N}_q^{\eta,2}(p_E^2)} + h.c. \right] \right\}, \quad (1)$$

where $\boldsymbol{Z}_q^{\eta,1}, \boldsymbol{N}_q^{\eta,2}$ involve Bessel-functions in dependence of bulk mass parameters and Yukawa couplings.

It is noteworthy that the bracket in (1) becomes independent of t in the limit $\eta \to 0$.

Therefore, we can obtain the same result by setting $t = t' = 1^-$ in the beginning while using the propagator solutions in the limit $\eta \to 0$.