Mitigation of the ϵ_K Fine-tuning Problem in the Randall-Sundrum Model

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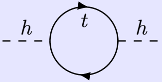
Introduction and Outline

Two major questions New Physics theories should answer

$$\mathcal{L}_{\text{EFT}}^{\text{SM}} = -\frac{1}{2} m_h^2 h^2 - \frac{v}{\sqrt{2}} \bar{u}_L Y_u u_R - \frac{v}{\sqrt{2}} \bar{d}_L Y_d d_R + \ldots + \frac{c_{ijkl}}{\Lambda_{\text{NP}}^2} \bar{q}_i q_j \bar{q}_k q_l + \ldots$$

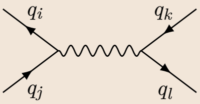
Naturalness principle



$\sim \Lambda_{\text{NP}}^2$

Hierarchical quark masses

$$m_u : m_c : m_t = 1 : 10^3 : 10^5$$
$$m_d : m_s : m_b = 1 : 10^2 : 10^3$$



The Minimal Randall-Sundrum Model offers an explanation

Goal

Minimize the bound on the M_{KK} scale

Idea

Extend the strong gauge sector in the Minimal Randall-Sundrum Model

Strongest bound from FCNC observable ϵ_K

Standard Model	Minimal RS Model
$\Lambda_{\text{NP}} \gtrsim 10^5$ TeV	$M_{\text{KK}} \gtrsim 8$ TeV $m_1^G \gtrsim 20$ TeV

Warped Extradimension: The Minimal Randall-Sundrum Model

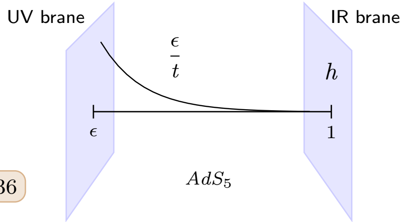
$$S_{5D}^{\text{RS}} = \int d^4x \int_{-r\pi}^{r\pi} dx_5 \sqrt{|G|} \left(-2M_{\text{Pl}(5)}^3 R_{(5)} - \Lambda_{(5)} - \delta(x_5)\, V_{\text{UV}} - \delta(|x_5| - r\pi)\, V_{\text{IR}} + \mathcal{L}_{\text{Fields}} \right)$$

Solution of Einstein equations

$$ds^2 = \frac{\epsilon^2}{t^2} \left(\eta_{\mu\nu}\, dx^\mu\, dx^\nu - \frac{1}{M_{\text{KK}}^2}\, dt^2 \right)$$

warp factor

$\epsilon \equiv e^{-L}$ $L = kr\pi \approx 36$



- Extra dimension: S^1/Z_2
- Bulk gauge group: $SU(3)_c \times SU(2)_L \times U(1)_Y$
- New Physics effects decouple with M_{KK}
- 5D fields live in the bulk except for the Higgs which is confined on the IR brane

$\left(M_{\text{Pl}(5)},\, k,\, 1/r \right) \sim \mathcal{O}(M_{\text{Pl}})$

$$G_\mu(x,t) = G_\mu^{(0)}(x)\chi_0^G(t) + \sum_{n=1}^{\infty} G_\mu^{(n)}(x)\chi_n^G(t)$$

$m_1^G = 2.45\, M_{\text{KK}}$

Solving the Hierarchy Problems and FCNCs

$$\mathcal{L}_{\text{EFT}}^{\text{RS}} = -\frac{1}{2}\,\big(\underbrace{\epsilon\, m_{h(5)}}_{}\overset{m_h}{}\big)^2 h^2 - \frac{1}{\sqrt{2}}\,\big(\overset{v}{\overbrace{\epsilon\, v_{(5)}}}\big)\,\bar{u}_L\, Y_u^{\text{eff}} u_R - \frac{1}{\sqrt{2}}\,\big(\overset{v}{\overbrace{\epsilon\, v_{(5)}}}\big)\,\bar{d}_L\, Y_d^{\text{eff}} d_R + \ldots$$

Warping strength

$$\epsilon = \frac{M_{\text{EW}}}{M_{\text{Pl}}} \approx 10^{-16}$$

Effective Yukawa matrix

$$Y_{q,ij}^{\text{eff}} = F(c_{Q_i})\, Y_{q,ij} F(c_{q_j})$$

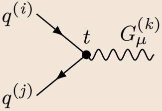
are responsible for the observed hierarchies

$[\,Y_{q,ij} \sim \mathcal{O}(1)$ and $c_{Q_i,q_i} \sim \mathcal{O}(1)]$

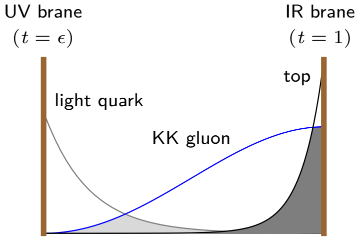
Profile overlap with IR brane

$$F(c) = \sqrt{\frac{1+2c}{1-\epsilon^{1+2c}}}$$

Suppression of FCNCs



$$\propto \int_\epsilon^1 dt\, \chi_i^q(t) \chi_j^q(t) \chi_k^G(t)$$



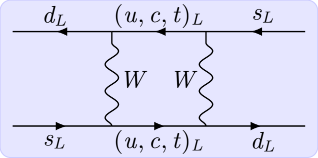
Observable ϵ_K

... measures indirect CP violation in K^0-$\bar{K}^0$ mixing

$$
\begin{aligned}
Q_1 &= (\bar{d}\gamma^\mu s)_L(\bar{d}\gamma_\mu s)_L \\
\tilde{Q}_1 &= (\bar{d}\gamma^\mu s)_R(\bar{d}\gamma_\mu s)_R \\
Q_4 &= (\bar{d}^\alpha \gamma^\mu s^\beta)_L(\bar{d}_L^\beta \gamma_\mu s^\alpha)_R \\
Q_5 &= (\bar{d}^\alpha \gamma^\mu s^\alpha)_L(\bar{d}^\beta \gamma_\mu s^\beta)_R
\end{aligned}
$$

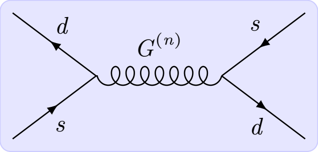
$$
\epsilon_K = \frac{\kappa_\epsilon e^{i\phi_\epsilon}}{\sqrt{2}(\Delta M_K)_{\text{exp}}} \, \mathrm{Im}\langle K^0 | \mathcal{H}_{\text{eff}} | \bar{K}^0 \rangle
$$

SM:



$$
|\epsilon_K^{\text{SM}}| = 1.81(28) \times 10^{-3}
$$

RS:



$Z^{(n)}, A^{(n)}, h$

$$
|\Delta\epsilon_K^{\text{RS}}| \propto C_1^{\text{RS}} + \tilde{C}_1^{\text{RS}} + \; 137 \; (C_4^{\text{RS}} + \frac{1}{3} C_5^{\text{RS}})
$$

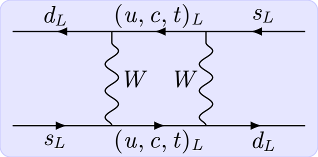
- ∘ renormalization group running
- ∘ hadronic matrix elements

Observable ϵ_K

... measures indirect CP violation in K^0-$\bar{K}^0$ mixing

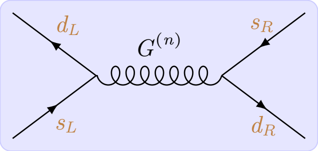
$$Q_1 = (\bar{d}\gamma^\mu s)_L(\bar{d}\gamma_\mu s)_L$$
$$\tilde{Q}_1 = (\bar{d}\gamma^\mu s)_R(\bar{d}\gamma_\mu s)_R$$
$$Q_4 = (\bar{d}^\alpha\gamma^\mu s^\beta)_L(\bar{d}_L^\beta\gamma_\mu s^\alpha)_R$$
$$Q_5 = (\bar{d}^\alpha\gamma^\mu s^\alpha)_L(\bar{d}^\beta\gamma_\mu s^\beta)_R$$

$$\epsilon_K = \frac{\kappa_\epsilon e^{i\phi_\epsilon}}{\sqrt{2}(\Delta M_K)_{\text{exp}}} \, \text{Im}\langle K^0|\mathcal{H}_{\text{eff}}|\bar{K}^0\rangle$$

SM: 

$$|\epsilon_K^{\text{SM}}| = 1.81(28) \times 10^{-3}$$

$$C_4^{\text{RS}} \approx -\frac{4\pi L\alpha_s}{M_{\text{KK}}^2}\frac{2m_d m_s}{Y_d^2 v^2} \Rightarrow M_{\text{KK}} \gtrsim 8\,\text{TeV}$$

RS: 

$Z^{(n)}, A^{(n)}, h$

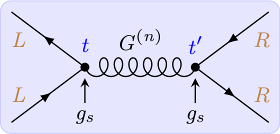
$$|\Delta\epsilon_K^{\text{RS}}| \propto C_1^{\text{RS}} + \tilde{C}_1^{\text{RS}} + 137\,\left(C_4^{\text{RS}} + \frac{1}{3}C_5^{\text{RS}}\right)$$

- ∘ renormalization group running
- ∘ hadronic matrix elements

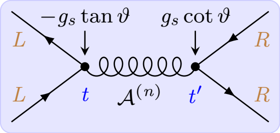
Mitigation through a Pseudo-Axial Gluon

Extend the bulk color gauge group to $SU(3)_D \times SU(3)_S$

$$\mathcal{L}_{\text{int}} \ni g_s \left(\bar{Q} \not{G}^a T^a Q + \bar{q}^c \not{G}^a T^a q^c \right) + g_s \left(-\tan\vartheta \, \bar{Q} \not{\mathcal{A}}^a T^a Q + \cot\vartheta \, \bar{q}^c \not{\mathcal{A}}^a T^a q^c \right)$$



$$\tilde{D}_G^{\mu\nu}(t,t') = \frac{\eta^{\mu\nu} L}{4\pi r M_{\text{KK}}^2} \Big[t_<^2 + \overbrace{\frac{1}{2L^2} - \left(t^2 \frac{1 - 2\ln t}{2L} + t \leftrightarrow t' \right)}^{\Delta F \le 1} \Big]$$



$$D_{\mathcal{A}}^{\mu\nu}(t,t') = \frac{\eta^{\mu\nu} L}{4\pi r M_{\text{KK}}^2} \Big[t_<^2 + \underbrace{c_0 + c_1 (t^2 + t'^2)}_{\Delta F \le 1} + c_2 \, t^2 t'^2 \Big]$$

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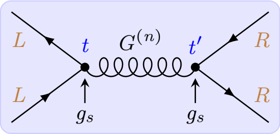

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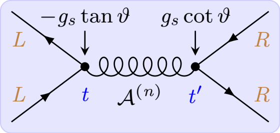
∘ cancels in C_4, C_5

∘ adds up in $C_1, \tilde{C}_1$



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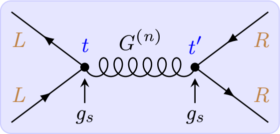
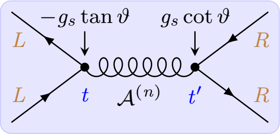


$$\tilde{D}^{\mu\nu}_G(t,t') = \frac{\eta^{\mu\nu} L}{4\pi r M_{\text{KK}}^2} \Big[t_<^2 + \overbrace{\frac{1}{2L^2} - \left(t^2 \frac{1 - 2\ln t}{2L} + t \leftrightarrow t' \right)}^{\Delta F \le 1} \Big]$$

- cancels in C_4, C_5
- adds up in $C_1, \tilde{C}_1$



$$D^{\mu\nu}_{\mathcal{A}}(t,t') = \frac{\eta^{\mu\nu} L}{4\pi r M_{\text{KK}}^2} \Big[t_<^2 + \underbrace{c_0 + c_1 (t^2 + t'^2)}_{\Delta F \le 1} + c_2 \, t^2 t'^2 \Big]$$

$c_2 \equiv c_2(b_\epsilon, b_1)$

UV BC: $\left. (\partial_t - b_\epsilon) \chi_n^{\mathcal{A}}(t) \right|_{t=\epsilon^+} = 0$

IR BC: $\left. (\partial_t + b_1) \chi_n^{\mathcal{A}}(t) \right|_{t=1^-} = 0$

$$\frac{C_4^{G+\mathcal{A}}}{C_4^G} \approx \frac{1}{2} \frac{b_\epsilon b_1}{b_\epsilon (2 + b_1) + 2 b_1 \epsilon}$$

Properties of the Pseudo-Axial Gluon

Breaking of $SU(3)_D \times SU(3)_S \to SU(3)_c$ determines boundary parameters b_ϵ, b_1

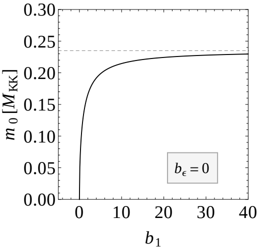
$$b_\epsilon \sim L\epsilon \frac{v_{\mathsf{UV}}^2}{M_{\mathsf{KK}}^2}$$

$$b_1 \sim L \frac{v_{\mathsf{IR}}^2}{M_{\mathsf{KK}}^2}$$

$$v_{\mathsf{UV}} \sim M_{\mathsf{Pl}}$$
$$v_{\mathsf{IR}} \sim v$$

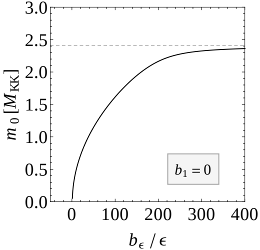
Three Scenarios

$v_{\mathsf{UV}} = 0, v_{\mathsf{IR}} \neq 0$



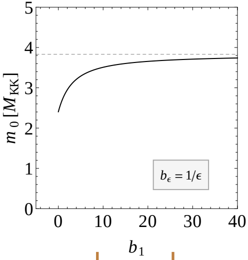
○ experimentally excluded

$v_{\mathsf{UV}} \neq 0, v_{\mathsf{IR}} = 0$



○ no Yukawa sector

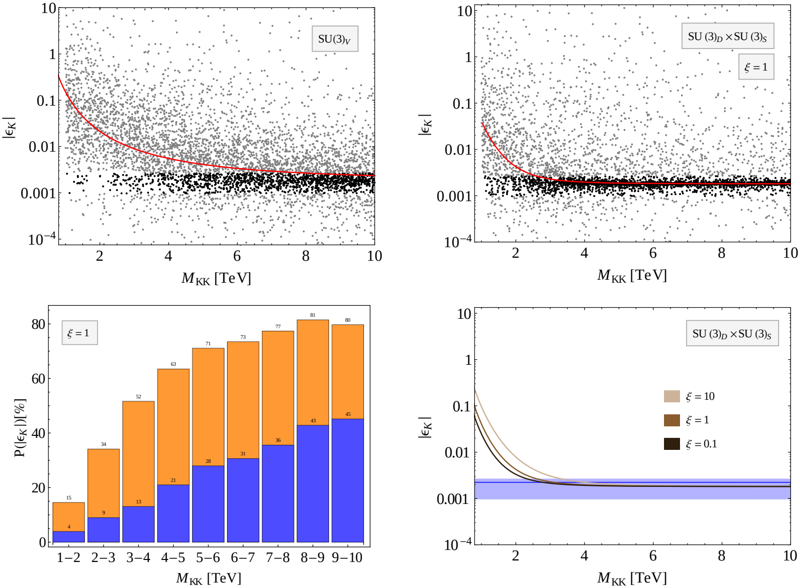
$v_{\mathsf{UV}} \neq 0, v_{\mathsf{IR}} \neq 0$



$$\frac{C_4^{G+\mathcal{A}}}{C_4^G} \approx \frac{1}{2}\frac{b_1}{2+b_1}$$

Determine b_1 from breaking of $SU(3)_D \times SU(3)_S$ at the IR brane

General Results with IR Boundary Parameter $b_1 = \xi L v^2 / M_{KK}^2$



Conclusion

- Implementation of an axial gluon leads to an additional v^2/M_{KK}^2 suppression of the mixed-chirality coefficients C_4 and C_5.
- The exact suppression factor depends on the boundary parameters b_ϵ, b_1 of the axial gluon.
- Only the scenario where $SU(3)_D \times SU(3)_S$ is broken to $SU(3)_c$ at both UV and IR branes is allowed.
- Therefore an enlarged Higgs sector localized at the UV and IR brane is needed.
- Using a parametrization of b_1 without considering some specific Higgs sector, the mitigation of the bound allows for $M_{\mathsf{KK}} \sim 2$-3 TeV concerning the scatter plots.

Thank you for your attention!

Extension of the Higgs Sector

Minimal and realistic approach needs at least the following field content:

scalars	$SU(3)_D$	$SU(3)_S$	$SU(2)_L$	$U(1)_Y$	localisation	vev
S	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	UV	v_S
H_l	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$	IR	v_l
H_u	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$-1/2$	IR	v_u
H_d	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{2}$	$1/2$	IR	v_d

IR boundary parameter: $$b_1 = \frac{\zeta g_s^2}{2N_c \sin^2\vartheta \cos^2\vartheta} L \frac{v^2}{M_{\text{KK}}^2} \qquad \zeta = \frac{v_u^2 + v_d^2}{v^2}$$

Reproduce correct W, Z masses

$$\zeta \lesssim 1$$

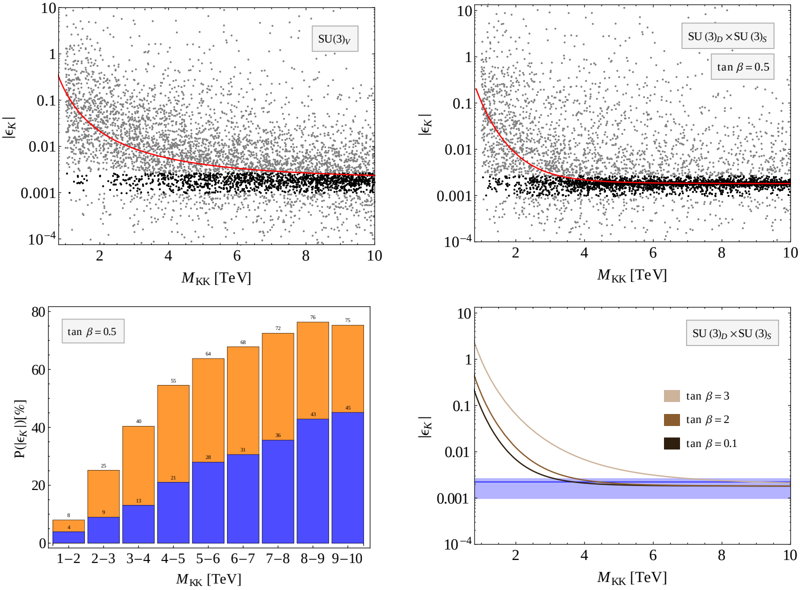
Relocalisation of righthanded quarks

$$F(c_{q_i}) \to \frac{\sqrt{N_c}\,v}{v_q} F(c_{q_i}) \equiv F(\tilde{c}_{q_i})$$

$$\theta = 45^\circ \qquad \frac{C_4^{G+\mathcal{A}}}{C_4^G} \approx \frac{1}{2}\left(1 + \tan^2\beta\right) L \frac{v^2}{M_{\text{KK}}^2} \qquad \tan\beta = \frac{v_u}{v_d}$$

Suppression sets in for $M_{\text{KK}} \gtrsim (1 + \tan^2\beta)$ TeV

Results

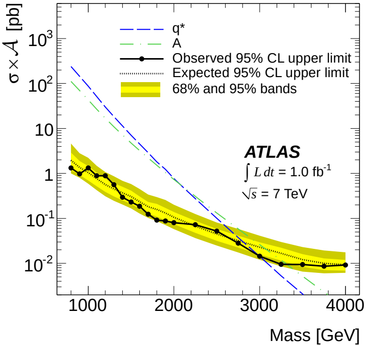

SU(3)$_V$
$|\epsilon_K|$
M_{KK} [TeV]
SU(3)$_D$ × SU(3)$_S$
tan $\beta = 0.5$
$|\epsilon_K|$
M_{KK} [TeV]
tan $\beta = 0.5$
P($|\epsilon_K|$)[%]
8
4
25
9
40
13
55
21
64
28
68
31
72
36
76
43
75
45
1−2
2−3
3−4
4−5
5−6
6−7
7−8
8−9
9−10
M_{KK} [TeV]
SU(3)$_D$ × SU(3)$_S$
tan $\beta = 3$
tan $\beta = 2$
tan $\beta = 0.1$
$|\epsilon_K|$
M_{KK} [TeV]

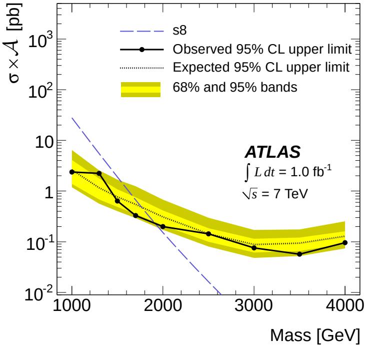
Conclusion

- Implementation of an axial gluon leads to an additional v^2/M_{KK}^2 suppression of the mixed-chirality coefficients C_4 and C_5.
- The exact suppression factor depends on the boundary conditions of the axial gluon.
- Only the scenario where $SU(3)_D \times SU(3)_S$ is broken to $SU(3)_c$ at both UV and IR branes is allowed.
- Considering one concrete Higgs sector, the relocalisation of the down-type quarks towards the IR brane, which counteracts the v^2/M_{KK}^2 suppression, introduces a dependence on the vev ratio $\tan\beta$.
- Still the bound on M_{KK} can be mitigated for small $\tan\beta \lesssim 2$.

Thank you for your attention!

Bounds on Axial Gluon and Octet Mass





Effects on other Observables

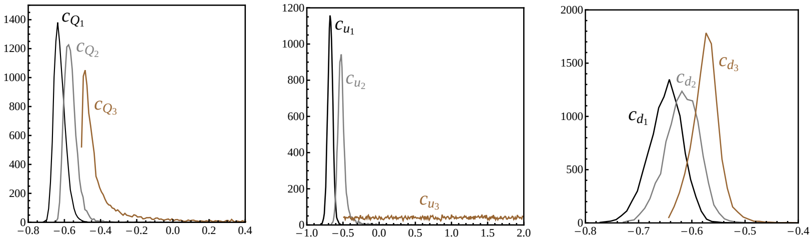
- S, T parameter: not directly affected by the modified strong sector and the relocalisation
- $Z \to b\bar{b}$ coupling: prefers small $\tan\beta$
- $\Delta B = 2$ observables: C_1 and $\tilde{C}_1$ are enhanced by $\cos^{-2}\theta$, $\sin^{-2}\theta$
- Experimental Bounds: Exclusion for masses below $m_1^{\mathcal{A}} \approx 2.45$ TeV

Generating a Physical Point

Compared to the SM, the RS model contains the additional parameters L, M_{KK}, $\boldsymbol{M}_{c_Q}$, $\boldsymbol{M}_{c_u}$ and $\boldsymbol{M}_{c_d}$.

Randomize $\bar{X} = (M_{\mathsf{KK}}, \boldsymbol{Y}_u, \boldsymbol{Y}_d, \boldsymbol{c}_Q, \boldsymbol{c}_u, \boldsymbol{c}_d)$, $Y_{q,ij} \in [0,3]$

Matching on $\bar{Y} = (m_u, m_c, m_t, m_d, m_s, m_b, A, \lambda, \bar{\rho}, \bar{\eta})$



Parameter counting concerning bulk mass and Yukawa matrices:

$$N_{\mathsf{phys}} = N_Y + N_c - (N_{G_Y} - N_{H_Y}) = (27, 10) \ni (9, 1)$$

Enlarged Higgs Sector: Field Content

Spontaneous breaking at both branes of

$$SU(3)_D \times SU(3)_S \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{Q_e}$$

leads to the following field content and series expansion at the

UV brane: 2 neutral singlets and 2 neutral octets

$$(S)_{\alpha\bar{\alpha}} = \left(v_0 + S^0_{(0)R} + iS^0_{(0)I}\right) \frac{\delta_{\alpha\bar{\alpha}}}{\sqrt{2N_c}} + \left(O^{0,a}_{(0)R} + iO^{0,a}_{(0)I}\right) T^a_{\alpha\bar{\alpha}} \ .$$

IR brane: 6 neutral and 3 charged singlets as well as 4 neutral and 2 charged octets

$$H_l = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\, \hat{S}^+ \\ \hat{v} + \hat{S}^0_R + i\hat{S}^0_I \end{pmatrix},$$

$$(H_u)_{\alpha\bar{\alpha}} = \begin{pmatrix} v_1 + S^0_{(1)R} + iS^0_{(1)I} \\ \sqrt{2}\, S^-_{(1)} \end{pmatrix} \frac{\delta_{\alpha\bar{\alpha}}}{\sqrt{2N_c}} + \begin{pmatrix} O^{0,a}_{(1)R} + iO^{0,a}_{(1)I} \\ \sqrt{2}\, O^{-,a}_{(1)} \end{pmatrix} T^a_{\alpha\bar{\alpha}},$$

$$(H_d)_{\alpha\bar{\alpha}} = \begin{pmatrix} \sqrt{2}\, S^+_{(2)} \\ v_2 + S^0_{(2)R} + iS^0_{(2)I} \end{pmatrix} \frac{\delta_{\alpha\bar{\alpha}}}{\sqrt{2N_c}} + \begin{pmatrix} \sqrt{2}\, O^{+,a}_{(2)} \\ O^{0,a}_{(2)R} + iO^{0,a}_{(2)I} \end{pmatrix} T^a_{\alpha\bar{\alpha}} \ .$$

Enlarged Higgs Sector: Potential Terms

UV Potential

$$V_{\mathsf{UV}} = -(\mu^{(S)})^2 S^{A\dagger} S^B \mathsf{Tr}[T^A T^B] + S^{A\dagger} S^B S^{C\dagger} S^D \boldsymbol{P}^{1,2}_{ABCD}(\lambda^{(S)}) + (e^{(S)} S^A S^B S^C \boldsymbol{R}_{ABC} + \mathsf{h.c.})$$

IR Potential

$$V_{\mathsf{IR}} = V^{(l)} + \sum_{q=u,d} \left[V^{(q)} + V^{(l,q)}_{\mathsf{mix}} \right] + V^{u,d}_{\mathsf{mix}} + V^{l,u,d}_{\mathsf{mix}},$$

where,

$$\begin{aligned}
V^{(l)} &= -\mu_l^2 |H_l^2| + \lambda_l |H_l|^4 \\
V^{(q)} &= -(\mu^{(q)})^2 S^{A\dagger} S^B \mathsf{Tr}[T^A T^B] + H_q^{i,A\dagger} H_q^{i,B} H_q^{j,C\dagger} H_q^{j,D} \boldsymbol{Q}^{1234}_{ABCD}(\lambda^q) \\
V^{(l,q)} &= c_1^{(q)} |H_l|^2 H_q^{i,A} H_q^{i,B} \mathsf{Tr}[T^A T^B] + c_2^{(q)} H_l^{i\dagger} H_l^j H_q^{j,A\dagger} H_q^{i,B} \mathsf{Tr}[T^A T^B] \\
&\quad + c_3^{(q)} \epsilon_{ik} \epsilon_{jl} H_l^{i\dagger} H_l^j H_q^{k,A\dagger} H_q^{l,B} \mathsf{Tr}[T^A T^B] \\
V^{(u,d)}_{\mathsf{mix}} &= H_u^{i,A\dagger} H_u^{i,B} H_d^{j,C\dagger} H_d^{j,D} \boldsymbol{Q}^{1,2,5,6}_{ABCD}(f) + H_u^{i,A\dagger} H_d^{i,B} H_d^{j,C\dagger} H_u^{j,D} \boldsymbol{Q}^{3,4,7,8}_{ABCD}(f) \\
&\quad + \epsilon_{ik} \epsilon_{jl} H_u^{i,A\dagger} H_u^{j,B} H_d^{k,C\dagger} H_d^{l,D} \boldsymbol{Q}^{9,10,11,12}_{ABCD}(f) \\
V^{(l,u,d)}_{\mathsf{mix}} &= \epsilon_{ik} H_l^{i\dagger} H_l^j H_u^{k,A\dagger} H_d^{j,B} \mathsf{Tr}[T^A T^B] + e_1 \epsilon_{ij} \epsilon_{kl} H_l^i H_u^j H_u^k H_d^l \boldsymbol{R}_{ABCD} \\
&\quad + e_2 \epsilon_{kl} H_l^{i\dagger} H_d^i H_d^k H_u^l \boldsymbol{R}_{ABCD} + \mathsf{h.c.}
\end{aligned}$$