Quantenfeldtheorie und Theoretische Elementarteilchenphysik

(exercise sheet 4)

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1 Generating functionals in ϕ^3 theory

The generating functional $Z[J]$ in the scalar ϕ^3 theory is given in the path integral formalism by

$$Z[J] = \exp\left[-i \int d^4z\, V\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)\right] Z_0[J], \tag{1}$$

with

$$Z_0[J] = Z_0[0]\, \exp\left[-\frac{i}{2}\int\int d^4x\, d^4y\, J(x)\, D_F(x,y)\, J(y)\right], \qquad V(\phi) = \frac{\lambda}{3!} m\, \phi^3, \tag{2}$$

where m is some mass scale and λ is a dimensionless coupling. Assuming that λ is small we can expand the generating functional such that

$$Z[J] = Z_0[J]\left(1 + \lambda Z_1[J] + \lambda^2 Z_2[J] + \mathcal{O}(\lambda^3)\right). \tag{3}$$

1.1 [8 points]

Calculate $Z_1[J]$ and $Z_2[J]$, draw the corresponding diagrams and classify them according to whether they are connected or disconnected (note that terms without any sources do not contribute and can be neglected). Here, the word "disconnected" means that the external points (sources) are not all connected with each other.

1.2 [2 points]

Now consider the logarithm of the generating functional defined by

$$W[J] = -i \ln Z[J], \tag{4}$$

and show that the diagrams at order λ^2 derived from $W[J]$ are connected. One can also generalize this procedure further to all orders in perturbation theory and prove that the functional $W[J]$ generates only connected Green's functions.