Quantenfeldtheorie und Theoretische Elementarteilchenphysik

(exercise sheet 2)

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1 Path integral for a freely moving particle

Show that the transition amplitude for a free particle with mass m moving in one dimension takes the form (set $\hbar = 1$)

$$\langle q_f, t_f | q_i, t_i \rangle = \left[\frac{m}{2\pi i (t_f - t_i)} \right]^{1/2} \exp \left[\frac{im}{2} \frac{(q_f - q_i)^2}{t_f - t_i} \right], \tag{1}$$

where $|q, t\rangle$ is a quantum state at time t and position q.

1.1 Hamiltonian method (3 points)

You should check that this result can be obtained from the Hamiltonian representation of the transition amplitude,

$$\langle q_f, t_f | q_i, t_i \rangle = \langle q_f | \exp\left[-iH(t_f - t_i)\right] | q_i \rangle, \tag{2}$$

where $H = p^2/2m$.

1.2 Path integral method (3 points)

Now you should use the path integral (Lagrangian) representations of the transition amplitude,

$$\langle q_f, t_f | q_i, t_i \rangle = \int \mathcal{D}q \, \exp \left[i \int_{t_i}^{t_f} dt \, L(q(t), \dot{q}(t)) \right], \tag{3}$$

where $L(q(t), \dot{q}(t)) = \frac{m}{2} \dot{q}^2(t)$ and the integration measure in the path integral representation is given by

$$\mathcal{D}q = \lim_{n \to \infty} \left(\frac{m}{2\pi i \tau} \right)^{(n+1)/2} \prod_{k=1}^{n} dq_k, \tag{4}$$

with $t_f - t_i$ ($t_f > t_i$) being divided into $n+1$ equal segments of $\tau : t_i \equiv t_0, t_1, t_2, \ldots, t_{n-1}, t_n, t_{n+1} \equiv t_f$ having the corresponding positions $q_i \equiv q_0, q_1, q_2, \ldots, q_{n-1}, q_n, q_{n+1} \equiv q_f$.
Remark: You can use the following formula for the successive Gaussian integrations

$$\int_{-\infty}^{\infty} dx \, \exp\left[a(x - x_1)^2 + b(x - x_2)^2 \right] = \left[\frac{-\pi}{a + b} \right]^{1/2} \exp \left[\frac{ab}{a + b} (x_1 - x_2)^2 \right] \tag{5}$$

where $a, b \in \mathbb{C}$ *and* $\operatorname{Re} a \leq 0, \operatorname{Re} b \leq 0$.