Quantenfeldtheorie und Theoretische Elementarteilchenphysik

(exercise sheet 1)

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1 Scalar ϕ^4 field theory

Consider the scattering of four uncharged scalar particles. The interaction Lagrangian (not normal ordered) is given for this process by

$$\mathcal{L}_I(x) = -\frac{\lambda}{4!}\,\phi^4(x)\ . \tag{1}$$

1.1 (5 points)

Use the Wick theorem and calculate all possible <u>connected</u> amplitudes up to second order in λ of

$$\lim_{T\to\infty(1-i\epsilon)} \langle 0|T\left\{\phi_I(x_1)\phi_I(x_2)\phi_I(x_3)\phi_I(x_4)\, e^{-i\int_{-T}^{T} dt\, H_I(t)}\right\}|0\rangle, \tag{2}$$

which is the vacuum expectation value of time-ordered products of field operators in the interaction picture and with $H_I(t) = -\int d^3y\, \mathcal{L}_I(t,\vec{y})$. Write them in terms of the Feynman propagator defined by

$$D_F(x-y) \equiv -i\langle 0|T\left\{\phi_I(x)\phi_I(y)\right\}|0\rangle\ . \tag{3}$$

Then draw the topologically different Feynman diagrams and explain the corresponding symmetry factor. *Remark: You can replace* $\lim_{T\to\infty(1-i\epsilon)}\int_{-T}^{T} dt \int d^3y$ *with* $\int d^4y$ *keeping in mind that for the time-component one has to integrate over a slightly distorted contour (but which is not relevant for this exercise).*

1.2 (1 point)

What would be different for the normal-ordered interaction Lagrangian $\mathcal{L}_I(x) = -\frac{\lambda}{4!} :\phi^4(x):$ (short statement) ?

2 Connected and disconnected diagrams (4 points)

Consider the unperturbed and perturbative parts of the (uncharged) scalar field theory

$$\mathcal{L}_0(x) = \frac{1}{2}(\partial_\mu\phi(x))^2, \qquad \mathcal{L}_I(x) = -\frac{m^2}{2} :\phi^2(x):, \tag{4}$$

where $m^2 > 0$. In perturbation theory, the connected two-point Green's function is given by

$$G^{(2)}(x_1,x_2) = \lim_{T\to\infty(1-i\epsilon)} \frac{\langle 0|T\left\{\phi_I(x_1)\phi_I(x_2)\exp[-i\int_{-T}^{T} dt\, H_I(t)]\right\}|0\rangle}{\langle 0|T\left\{\exp[-i\int_{-T}^{T} dt\, H_I(t)]\right\}|0\rangle]}, \tag{5}$$

where ϕ_I denotes an interaction picture field operator and $H_I(t) = -\int d^3y\, \mathcal{L}_I(t,\vec{y})$. Use Wick's theorem to verify explicitly that the disconnected graphs in the numerator and denominator cancel for the diagrams with up to three vertex insertions (you can apply the remark of exercise 1.1).